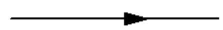
SECTION A: OPTICS

Light is a form of energy that travels in a straight line.
Some objects produce light on their own and these are called **luminous objects** e.g sun, fire worms, fire fly .
Most objects we see don't produce light on their own but reflect it from luminous objects and these are called **non-luminous objects** e.g. moon, the stars, etc.
Some objects do not allow light to go through them and these are called **opaque objects** e.g. wood, wall, people, etc.
Some of them allow most of the light to go through them and these are called **transparent objects** e.g. glass, clear water, clear polythene.
Other objects allow some light to go through them and these are called **translucent objects** e.g. paper, bathroom glasses, tinted glass, etc.

RAYS AND BEAMS

A ray is the direction of the path taken by light.
It is indicated by a straight line with an arrow on it.

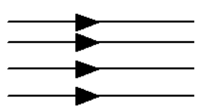


A beam is a collection of light rays.

OR: A beam is a stream of light energy.

Types of beams

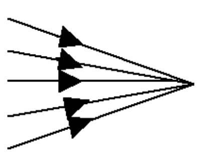
a) Parallel beam



Rays are parallel to each other.

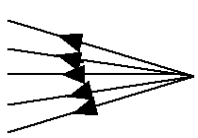
This is obtained from light from a distant source and search lights.

b) Convergent beam



Rays from different directions meet at a common point.
E.g. light behind a convex lens after passing through it.

c) Divergent beam



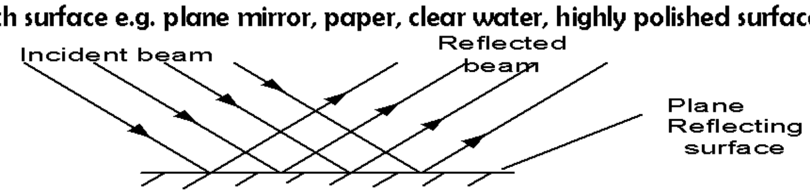
Rays start from a common point and separate into different directions.
E.g. light from a torch and car lights.

REFLECTION OF LIGHT

Reflection is the bouncing of light as it strikes a reflecting surface.

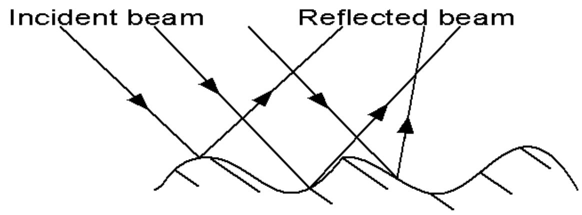
Types of reflection

a) **Regular reflection.** Here, an incident parallel beam is reflected as a parallel beam when light falls on a smooth surface e.g. plane mirror, paper, clear water, highly polished surfaces.

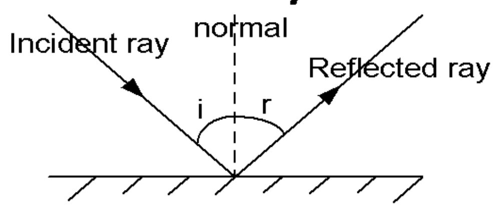


b) **Irregular (diffuse) reflection**

Here an incident parallel beam is reflected in different directions when light falls on a rough surface e.g. iron sheets, unclear water, etc.



Reflection from a plane mirror



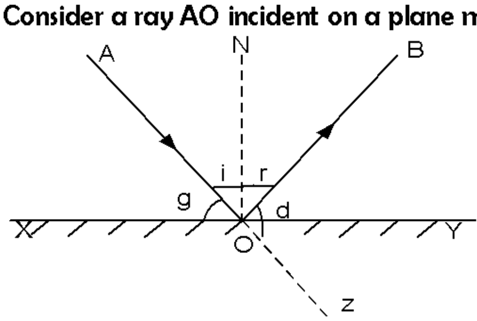
i = angle of incidence (angle between the normal and incident ray)
r = angle of reflection (angle between the normal and reflected ray)

LAWS OF REFLECTION

Law 1: The incident ray, the reflected ray and the normal at the point of incidence all lie on the same plane.
Law 2: The angle of incidence is equal to the angle of reflection.

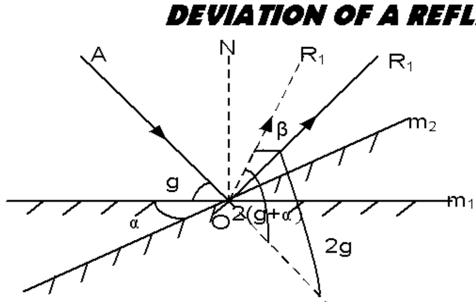
Deviation of light by a plane mirror

Consider a ray AO incident on a plane mirror M at a glancing angle g



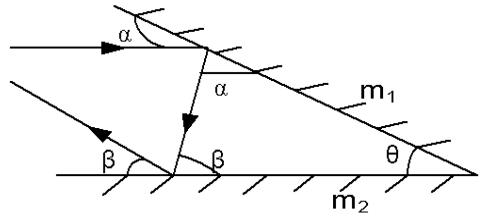
$$d = < BOY + < ZOY$$
$$< ZOY = g \ (vertically\ opposite)$$
$$< BOY = 90 - r$$
$$but\ i = r \ (law\ of\ refelction)$$
$$< BOY = 90 - i$$
$$but\ i = 90 - g$$
$$\therefore < BOY = 90 - (90 - g) = g$$
$$d = g + g$$
$$d = 2g$$

DEVIATION OF A REFLECTED RAY BY A ROTATED MIRROR



- A constant ray OA is incident onto a plane mirror in position m_1 and is reflected along OR_1
- The glancing angle$= g$ and deviation d caused$= 2g$
- When the mirror is turned through an angle α, the new glancing angle is $(g + \alpha)$ and new deviation$2(g + \alpha)$
- The angle β through whoich the ray is rotated is $\beta = 2(g + \alpha) - 2g$
  $$\beta = 2\alpha$$

DEVIATION BY SUCCESSIVE REFLECTION AT TWO INCLINED MIRRORS



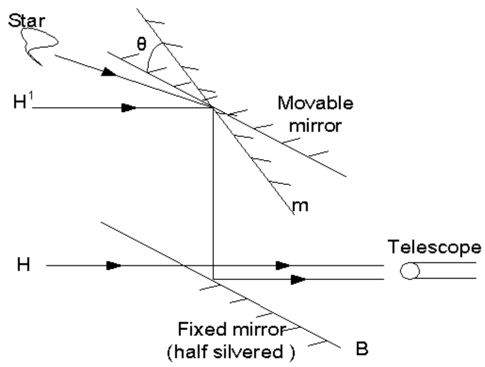
- A ray is incident onto a plane mirror m_1 and is reflected
- The glancing angle$=\alpha$ and deviation d caused$=2\alpha$
- When the ray is reflected at mirror m_2 , the new glancing angle is β and new deviation2β
- The net deviation, d $=2(\alpha+\beta)$ clockwise

$$but\ \beta+\theta+\alpha=180$$
$$\beta+\alpha=180-\theta$$
$$d=2(180-\theta)$$
$$d=(360-2\theta)\ clockwise$$

Or $360-(360-2\theta)=2\theta\ anticlockwise$

THE SEXTANT

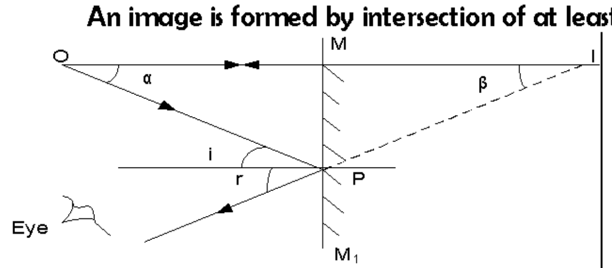
This is an instrument which is used for measuring the angle of elevation of heavenly bodies such as the star and the sun



- The setup is as above. B is half silvered fixed mirror while m can rotate
- First rotate m until the image of horizon H' is seen to coincide with horizon H
- At this point m and B are parallel, note position of mirror
- M is now rotated until the image of the star is seen to coincide with the horizon H
- The angle of rotation θ is measured. The angle of deviation is 2 θ

Formation of images by a plain mirror

An image is formed by intersection of at least two rays.



Consider an object O placed infront of a plane mirror. Rays of light form O are reflected form the mirror and appear to come form I. I is the virtual image of O

$$<i=<r\ \ (laws\ of\ reflection\ of\ light)$$
$$<i=<\alpha\ \ (alternate\ angles)$$
$$<r=<\beta\ \ (corresponding\ angles)$$
$$\Rightarrow\quad <\alpha=<\beta$$

Since side MP is common to both triangles ΔOMP and ΔIMP, the triangles are congreguent

Hence $OM=MI$

The image is as far behind the mirror as the object is infront

PROPERTIES OF IMAGES FORMED BY A PLANE MIRROR.

1. The images are the same distance behind the mirrors as the distance of the object in front of the mirror.
2. The images have the same size as the object.
3. The images are erect (upright)
4. The images are laterally inverted (rotated through 180° in the mirror).
5. The images are virtual (they cannot be formed on the screen).

THE REAL AND VIRTUAL IMAGES

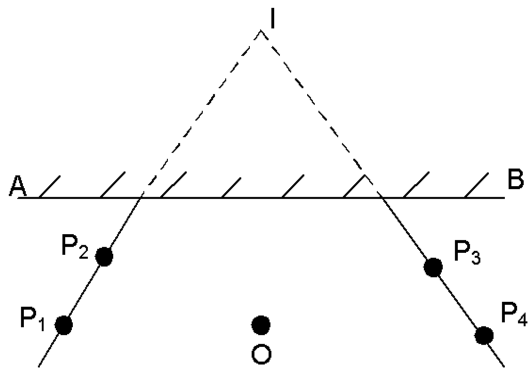
A real image

It is one which can be formed on the screen and is formed by the actual intersection of light rays e.g. images formed by concave mirror and convex lenses.

A virtual image

It is one that cannot be formed on a screen and is formed by the apparent intersection of light rays e.g. images formed by plane mirrors, concave lenses and convex mirrors.

LOCATION OF AN IMAGE ON PLANE MIRRORS

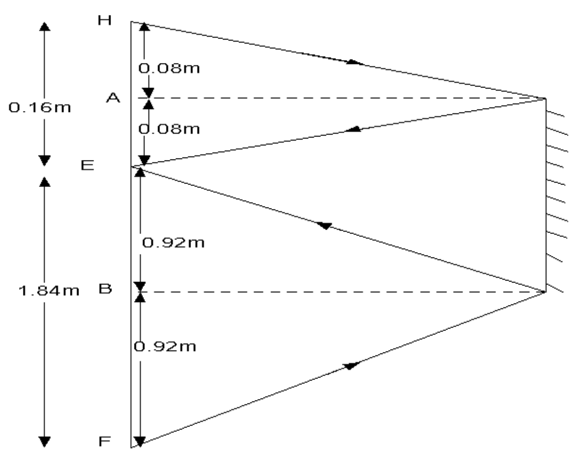


- An object pin O is placed in front of a plane mirror AB, on a white sheet of paper.
- Looking from side A of the mirror, two pins P_1 and P_2 are placed so that they look to be in line with the image of the pin O.
- The experiment is repeated with pins P_3 and P_4 on side B.
- The pins and the mirror are removed and lines drawn through the pin marks P_1P_2 and P_3P_4 to meet at I. I is the position of the image.

Minimum vertical length of a plane mirror

A man 2m tall whose eye level is 1·84m above the ground looks at his image in a Vertical mirror. What must be the minimum vertical length of the mirror so that the man can see the whole of himself **completely** in the mirror ?

Solution



Rays from the top of the man are reflected from the top of the mirror and are incident in the man's eyes (point E is the man's eye level)

Since HA = AE then,

$AE = \frac{1}{2}x \ \ 0{\cdot}16 \ = \ 0{\cdot}08m$

Similarly $EB \ = \ BF.$

Thus $EB = \frac{1}{2}x \ \ 1{\cdot}84 \ = \ 0{\cdot}92m$

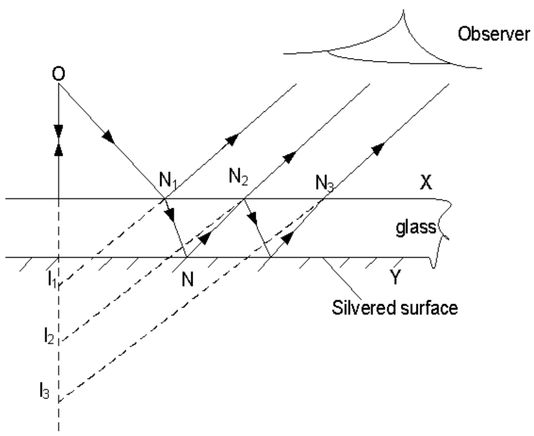
The minimum length of the mirror = $AE + EB$

$= \ 0{\cdot}08 \ + \ 0{\cdot}92$

$= \ 1m$

Hence the minimum length of the mirror is half the height of the object

FORMATION OF MULTIPLE IMAGES IN THICK PLANE MIRROR



- A thick plane has two plane surfaces say X and Y, reflection takes place at the two surfaces.
- The reflection at N_1 leads to the formation of image I_1
- The transmitted light is reflected at the silvered surface N, it ndergoes partial reflection and transmission at N_2
- The transmitted light appears to originate from I_2
- The successive internal reflections will lead to multiple images.

Note:

(i) Thick mirror forms multiple images and its distant images are faint. Multiple images is due to multiple images is due to successive reflections and faint images is due the energy absorbed at each reflection

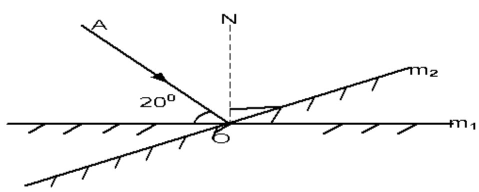
(ii) The disadvantages of using plane mirrors as reflectors of light in optical instruments such as submarine periscopes are overcome by using reflecting prisms.

COMPARISION OF PLANE MIRRORS AND REFLECTING PRISMS.

(i) Un like in prisms, plane mirrors produce multiple images
(ii) The silvering in plane mirrors wears out with time while no silvering is required in prisms
(iii) Unlike in prisms, plane mirrors exercise loss of brightness when reflection occurs at its surface.

EXERCISE: 1

1. What is meant by reflection of light?
2. State the laws of reflection of light
3. Distinguish between regular and diffuse reflection of light
4. Show with the aid of a ray diagram that when a ray of light is incident on a plane
5. mirror, the angle of deviation of a ray by the plane surface is twice the glancing angle.
6. Derive the relation between the angle of rotation of a plane mirror and the angle of deflection of a reflected ray, when the direction of the incident ray is constant.
7. An incident ray of light makes an angle of **20**° with the plane mirror in position m_1,as shown below



Calculate the angle of reflection, if the mirror is rotated through **6**° to position m_2 while the direction of the incident ray remains the same.

8. **(i)** Show that an incident ray of light reflected successively from two mirrors inclined at an angle θ to each other is deviated through an angle **2**θ.
(ii) Name one application of the result in **7(i)** above.

9. Describe how a sextant is used to determine the angle of elevation of a star.

10. Show that the image formed in a plane mirror is as far behind the mirror as the object is in front

11. State the characteristics of images formed by plane mirrors.

12. (i) What is meant by **No parallax method** as applied to location of an image?

(ii) Describe how the position of an image in a plane mirror can be located

13. Show that for a man of height, **H**, standing upright the minimum length of a vertical plane mirror in which he can see the whole of him self completely is $\frac{H}{2}$.

14. With the aid of a ray diagram, explain how a thick plane mirror forms multiple images of an object.

15. Give three reasons for using prisms rather than plane mirrors in reflecting optical instruments.

REFLECTION IN CURVED MIRRORS

CURVED MIRRORS ((Spherical mirrors)

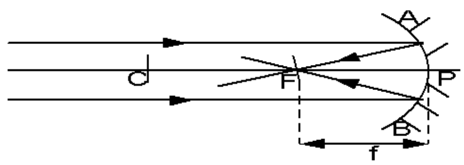
Curved mirrors are mirrors whose surfaces are obtained from a hollow transparent sphere.

There are two types of curved mirrors;

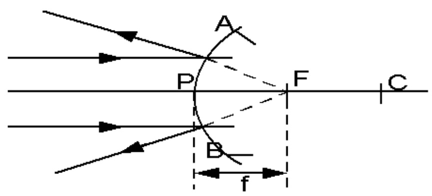
i) Concave mirror (converging mirror)

ii) Convex mirror (diverging mirror)

Concave (Converging) mirror: it is part of the sphere whose centre **C** is in front of its reflecting surface.



Convex (Diverging) mirror: it is part of the sphere whose centre **C** is behind its reflecting surface.



Where;

P is the pole of the mirror

F is the principal focus (focal point)

C is the centre of curvature

f is the focal length

r is the radius of curvature.

APB – Aperture

PFC – Principal axis

Terms used in Curved mirrors

Definitions

1. **Centre of curvature C:** it is the centre of the sphere of which the mirror forms part.
2. **Radius of curvature r:** it is the radius of the sphere of which the mirror forms part.
3. **Pole of the mirror:** it is the mid-point (centre) of the mirror surface.
4. **Princpal axis CP**: it is the line that passes through the centre of curvature and the pole of the mirror.
5. **Secondary axis:** line through the center of a thin lens or through the center of curvature of a concave or convex mirror other than the principal axis of the lens or mirror
6. **Paraxial rays:** These are rays close to the principal axis and make small angles with the mirror axis.
7. **Marginal rays:** These are rays furthest from the principal axis of the mirror.
8. **(i) Principal focus "F" of a concave mirror:** it is a point on the principal axis where paraxial rays incident on the mirror and parallel to the principal axis converge after reflection by the mirror.

   A concave mirror has a real (in front) principal focus.

   (ii). Principal focus "F" of a convex mirror: it is a point on the principal axis where paraxial rays incident on the mirror and parallel to the principal axis appear to diverge from after reflection by the mirror

   A convex mirror has a virtual (behind) principal focus.
9. **(i) Focal length "f" of a concave mirror:** it is the distance from the pole of the mirror to the point where paraxial rays incident and parallel to the

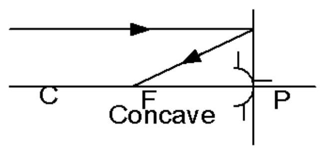
principal axis converge after reflection by the mirror.

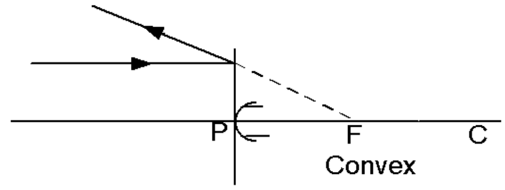
(ii) Focal length "f" of a convex mirror: it is the distance from the pole of the mirror to the point where paraxial rays incident and parallel to the principal axis appear to diverge from after reflection by the mirror.

10. Aperture of the mirror: it is the length of the mirror surface.

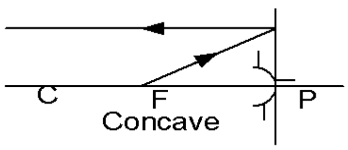
GEOMETRICAL RULES FOR THE CONSTRUCTION OF RAY DAIGRAMS(TO LOCATE IMAGE POSITIONS)

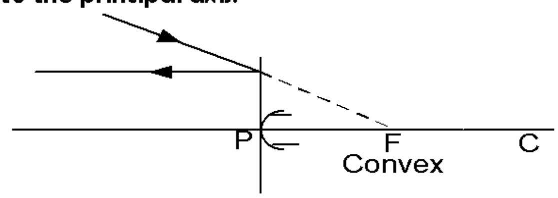
1. Rays are always drawn from the top of the object.
2. A ray parallel to the principal axis is reflected through the principal focus.



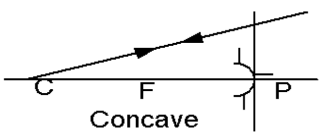


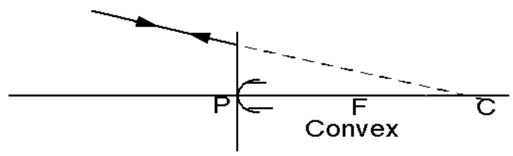
3. A ray through the principal focus is reflected parallel to the principal axis.





4. A ray through the centre of curvature is reflected along its own path.





5. Rays incident to the pole are reflected back, making the same angle with the principal axis.
6. At least two rays are used i.e. (1 and 2) or (1 & 3). Their point of intersection is where the image is, and it is always the top of the image.

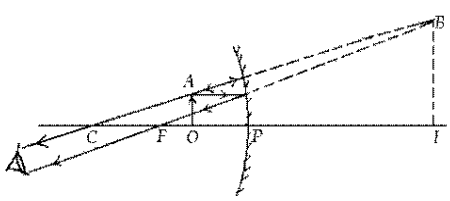
NOTE:

(i) The normal due to reflection at the mirror surface at any point must pass through the centre of curvature.

(ii) The image position can be located by the intersection of two reflected rays initially coming from the object.

IMAGES FORMED BY A CONCAVE MIRROR

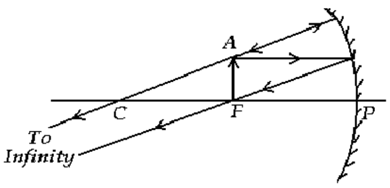
The nature of the image formed by a concave mirror is either real or virtual depending on the object distance from the mirror as shown below;



Object between F and P the image is

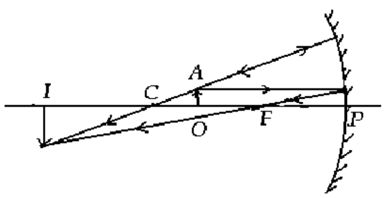
1) Behind the mirror
2) Virtual
3) Erect
4) Magnified

The property of a concave mirror to form erect, virtual and a magnified image when the object is nearer to the mirror than its focus makes it useful as a shaving mirror and also used by dentists for teeth examination.



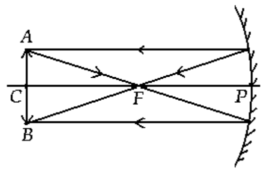
Object at F the image is

1) at infinity, virtual and upright



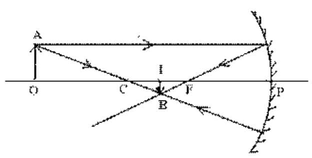
Object between F and C the image is

1) Beyond C
2) Real
3) Inverted
4) Magnified



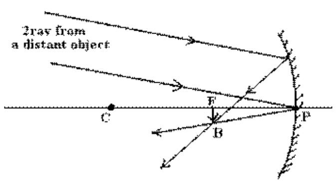
Object at C the image is

1) At C
2) Real
3) Inverted
4) Same size as the object



Object beyond C the Image is

1) Between C and F
2) Real
3) Inverted
4) Diminished



Object at infinity the image is

1) At F
2) Real
3) Inverted
4) Diminished

USES OF CONCAVE MIRRORS

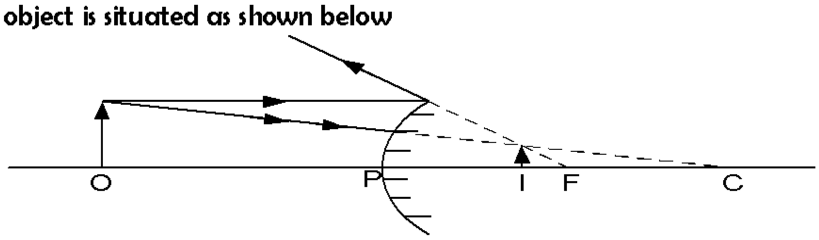
(i) They are used as shaving mirrors.
(ii) They are used by dentists for teeth examination.
(iii) They are used as solar concentrators in solar panels.
(iv) They are used in reflecting telescopes, a device for viewing distant objects
(v) They are used in projectors, a device for showing slides on a screen.

Advantage

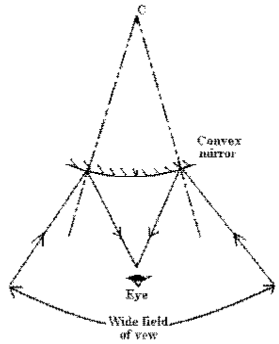
It forms magnified and erect images

IMAGES FORMED BY A CONVEX MIRROR

The image of an object in a convex mirror is erect, virtual, and diminished in size no matter where the object is situated as shown below



In addition to providing an erect image, convex mirrors have got a wide field of view as illustrated below.



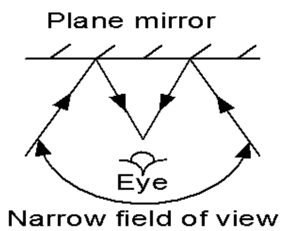
Note. A convex mirror can form real images if it receives converging rays targeting any point between Focal point &and its optical centre like in the above case(If the screen was placed at the position of the eye above a real image will be formed on it)

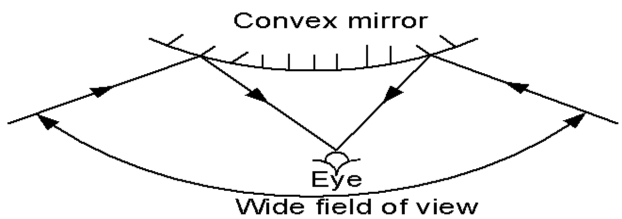
Convex mirrors

(i) They are used as driving mirrors. This is because they always form erect images and have a wide field of view.
(ii) Used in super markets to observe the activities of customers
(iii) Used in security check points to inspect under vehicles

Advantages of convex mirrors over plane mirrors

i) They have a wide field of view
ii) They form erect images



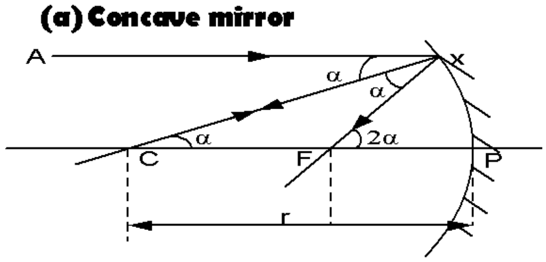


Disadvantages of convex mirrors

It diminished images ,giving a wrong impression to drivers that the object behind is very far

Relation between focal length and radius of curvature

(a) Concave mirror



A ray AX close and parallel to the principal axis is reflected through the principal focus F

$$FP = focal\ lenght\ (f)$$

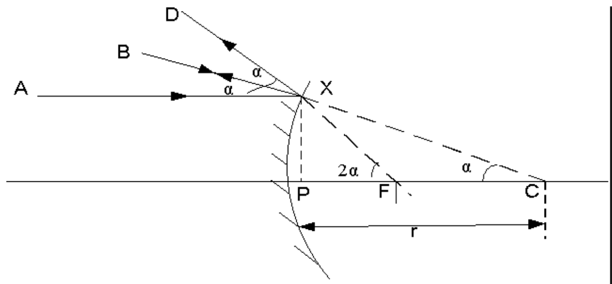
If C is the centre of curvature, then CP is the radius of the mirror cx

$$< AXC =< CXF = \alpha(law\ of\ reflcetion)$$
$$< XCP =< AXC = \alpha(alternate\ angles\ )$$
$$FC = FX$$

For AX close to CP

$$FX \approx FP$$
$$\therefore CF = FP$$
$$2\,FP = CP = r$$
$$r = 2f$$

(b) Convex mirror



A ray AX close and parallel to the principal axis is reflected through the principal focus F

$$FP = focal\ lenght\ (f)$$

If C is the centre of curvature, then CP is the radius of the mirror cx

$$< AXB =< BXD = \alpha(law\ of\ reflcetion)$$
$$< AXB =< XCP = \alpha(alternate\ angles\ )$$
$$FC = FX$$

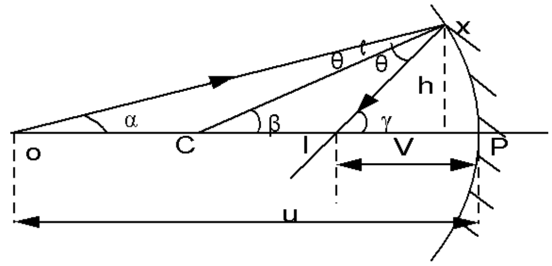
For AX close to CP

$$FX \approx FP$$
$$\therefore CF = FP$$
$$2\ FP = CP = r$$
$$r = 2f$$

MIRROR FORMULAR

(a) Concave mirror

Consider a point object O on the principal axis of a concave mirror



From triangle OXC; $\theta = \beta - \alpha$..................(1)

From triangle CXI; $\theta = \gamma - \beta$..................(2)

From eqn1 and eqn2

$$\beta - \alpha = \gamma - \beta$$
$$2\beta = \gamma + \alpha \text{...............(3)}$$

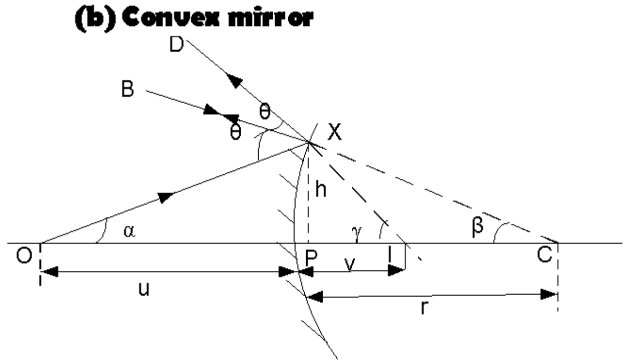
$for\ small\ angles\ in\ radians\ tan\alpha \approx \alpha,$ $tan\beta \approx \beta, tan\gamma \approx \gamma$

$$\frac{2\ h}{CP} = \frac{h}{IP} + \frac{h}{OP}$$
$$\frac{2}{r} = \frac{1}{v} + \frac{1}{u}$$

But

$$r = 2f$$
$$\frac{2}{2f} = \frac{1}{v} + \frac{1}{u}$$
$$\boxed{\frac{1}{f} = \frac{1}{v} + \frac{1}{u}}$$

(b) Convex mirror



From triangle OXC; $\theta = \beta + \alpha$..................(1)

From triangle CXI; $2\theta = \gamma + \ \alpha$..................(2)

From eqn1 and eqn2

$$\beta - \alpha = \gamma - \beta$$

$$2(\beta + \alpha) = \gamma + \alpha$$
$$2\beta = \gamma - \alpha \text{...............(3)}$$

$for\ small\ angles\ in\ radians\ tan\alpha \approx \alpha,$ $tan\beta \approx \beta, tan\gamma \approx \gamma$

$$\frac{2\ h}{CP} = \frac{h}{IP} - \frac{h}{OP}$$
$$\frac{2}{-r} = \frac{1}{-v} - \frac{1}{u}$$

But $r = 2f$

$$\frac{2}{2f} = \frac{1}{v} + \frac{1}{u}$$
$$\boxed{\frac{1}{f} = \frac{1}{v} + \frac{1}{u}}$$

Where

u =object distance

v = image distance

f = focal length

r = radius of curvature

Sign convention

- Distances of real objects and real images are positive ie u and v for real objects and real images are positive.
- Distances of virtual objects and virtual images are negative ie u and v for virtual objects and virtual images are negative.
- Focal length f, for a concave mirror is positive and negative for a convex mirror.

LINEAR MAGNIFICATION

It is defined as the ratio of the image height to object height.

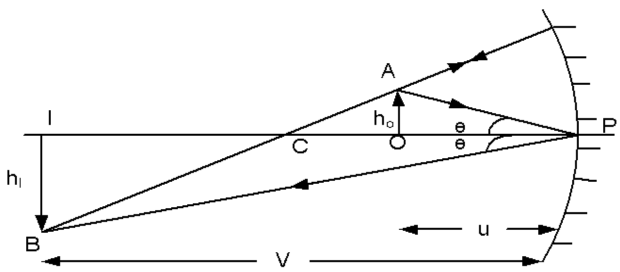
$$m = \frac{height\ image}{height\ object}$$

Magnification can also be obtained by determining the ratio of distance of the image from the mirror (v) to the distance of the object from the mirror (u)

$$m = \frac{image\ distance\ (v)}{object\ distance\ (u)}$$

PROOF

Consider the incidence of ray **AP** on to the pole of a concave mirror from an object of height **h** placed a distance, **u,** from the mirror and then reflected back making the same angle with the principal axis to form an image of height **h_I**, located at distance, **v**, from the mirror as shown



Ray **AP** makes an angle θ with the normal **OP**, then,

From Δ OAP, $tan\ \theta = \frac{h_o}{u}$ ---------------(i)

From Δ IPB, $tan\ \theta = \frac{h_1}{v}$.............................(ii)

Equating equation (i) and (ii) gives. $\frac{h_o}{u} = \frac{h_1}{v}$

Thus magnification, $m, = \frac{v}{u} = \frac{h_I}{h_o}$

NOTE:

(i) No signs need be inserted in the magnification formula.

(ii) Using the mirror formula, a connection relating magnification to the focal length of the mirror with either the object distance or the image distance can be established.

Example

1. An object 1cm tall is placed 30cm in front of a concave mirror of focal length 20cm. find ;

(i) The position of the image

(ii) The size of the image formed

(iii) The magnification of the image

Solution

ho= 1cm, hi=?, u=30cm, v=?,
f=20cm (concave mirror)

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{v} = \frac{1}{20} - \frac{1}{30}$$

$$\frac{1}{v} = \frac{1}{60}$$

V =60cm
Positive sign means the image is real (60cm in front of the mirror)

M = $\frac{h_i}{h_o} = \frac{v}{u}$

$$\frac{h_i}{1} = \frac{60}{30}$$

h_i =2cm

m = $\frac{v}{u}$ = $\frac{60}{30}$
m =2
Or
M = $\frac{h_i}{h_o}$ = $\frac{2}{1}$
M = 2

2. An object 10cm tall is placed 30cm in front of a convex mirror of focal length 20cm. Find;

(i) The position

(ii) The size of the image formed.

Solution

ho= 10cm, hi=?, u=30cm, v=?,
f= - 20cm (convex mirror)

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{v} = \frac{1}{-20} - \frac{1}{30}$$

$$\frac{1}{v} = \frac{-5}{60}$$

V = - 12cm
Negative sign means the image is virtual (12cm behind the mirror)

M = $\frac{h_i}{h_o} = \frac{v}{u}$

$$\frac{h_i}{10} = \frac{12}{30}$$

h_i =4cm

3. A small object is placed on the principal axis of a convex mirror of curvature of 20cm. Determine the position of the image when the object is 15cm from the mirror

Solution

u=15cm, v=?, r=20cm, f=$\frac{r}{2}$=$\frac{20}{2}$

f= - 10cm (convex mirror)

$\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$

$\frac{1}{v}=\frac{1}{-10}-\frac{1}{15}$

$\frac{1}{v}=\frac{-5}{30}$

V = - 6cm

Negative sign means the image is virtual (6cm behind the mirror)

4. A concave mirror with radius of curvature 40cm forms an image of real object which is placed 25cm from the mirror

(a) What is the focal length of the mirror

(b) Calculate the distance of the image from the mirror and give its nature

Solution

$f=\frac{r}{2}$

$f=\frac{40}{2}$

$f=20cm$

$\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$

$\frac{1}{v}=\frac{1}{20}-\frac{1}{25}$

$\frac{1}{v}=\frac{1}{100}$

$v\ =100cm$

5. An object is placed 10cm in front of a concave mirror of focal length 15cm. Find the image position and magnification.

Solution:

$\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$

$\frac{1}{v}=\frac{1}{15}-\frac{1}{10}$

$v\ =-30cm$

The negative sign implies that the image formed is **virtual** and it is formed 30cm from the mirror

Magnification, M= $\frac{v}{u}$

$m=\frac{30}{10}=3$

6. The image of an object in a convex mirror is 6cm from the mirror. If the radius of curvature of the mirror is 20cm, find the object position and the magnification.

Solution:

For a convex mirror, $f=\frac{r}{2}$

$f=\frac{-20}{2}$

$f=-`10cm$

v = – 6cm (The image in a convex mirror is always virtual)

Using the mirror formula $\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$

$\frac{1}{u}=\frac{1}{-10}-\frac{1}{-6}$

$u\ =15cm$

∴ **Magnification,** M= $\frac{v}{u}=\frac{6}{15}=0.4$

7. Show that an object and its image coincide in position at the centre of curvature of a concave mirror. Hence find the magnification produced in this case.

Solution

At the centre of curvature of a concave mirror, object distance $u\ =\ r$ and $r=2f$ where r is the radius of curvature of the mirror.

$\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$

$\frac{2}{r}=\frac{1}{r}+\frac{1}{v}$

$\frac{1}{v}=\frac{1}{2r}-\frac{1}{r}$

$v\ =r$

Thus the image is also formed at the centre of curvature and therefore it coincides in position with its object.

Hence, M= $\frac{v}{u}=\frac{r}{r}=1$

∴ The object and its image are of the same size in this case.

RELATIONSHIP CONNECTING m, v and f

From $\frac{1}{f}=\frac{1}{v}+\frac{1}{u}$

multiplying all through by v

$$\frac{v}{f}=\frac{v}{v}+\frac{v}{u}$$

$$\boxed{m=\frac{v}{f}-1}$$

RELATIONSHIP CONNECTING m, u and f

From $\frac{1}{f}=\frac{1}{v}+\frac{1}{u}$

multiplying all through by u

$$\frac{u}{f}=\frac{u}{v}+\frac{u}{u}$$

$$\boxed{\frac{1}{m}=\frac{u}{f}-1}$$

Examples

1. A small objects placed infront of a spherical mirror gives a real image and 4 times the size of the object. When the object is moved 10cm towards the mirror a similarly magnified virtual image is formed. Find the focal length of the mirror

Solution

$\frac{1}{m}=\frac{u}{f}-1$

$u_1=\left(\frac{1}{m_1}+1\right)f$.............(1)

$u_2=\left(\frac{1}{m_2}+1\right)f$.............(2)

For object moved towards the mirror, $u_2<u_1$ then

$u_1-u_2=10$................(3)

$f=\frac{u_1-u_2}{\frac{1}{m_1}-\frac{1}{m_2}}$

$f=\frac{10}{\frac{1}{4}-\frac{1}{-4}}=20cm$

2. A concave mirror forms a real image which is 3 times the linear size of the real object. When the object is displaced a distance d, the real image formed is now 4 times its linear size of the object. If the distance between the two images position is 20cm. find
   (i) focal length of the mirror
   (ii) Distance d

Solution

$m=\frac{v}{f}-1$

$v_1=(m_1+1)f$.............(1)

$v_2=(m_2+1)f$.............(2)

For $m_2>m_1, v_2>v_1$ then

$v_2-v_1=20$................(3)

$f=\frac{v_2-v_1}{m_2-m_1}$

$f=\frac{20}{4-3}=20cm$

For $m_2>m_1, u_1>u_2$ then

$\frac{1}{m}=\frac{u}{f}-1$

$u_1=\frac{f}{m_1}+1$.............(1)

$u_2=\frac{f}{m_2}+1$.............(1)

$d=u_1-u_2$

$d=\frac{f}{m_1}-\frac{f}{m_2}$

$d=\frac{20}{3}-\frac{20}{4}$

$\boldsymbol{d=1.67cm}$

3. A concave mirror forms on a screen a real image of three times the size of the object. The object and screen are then moved until the image is five times the size of the object. If the shift of the screen is **30cm,** determine the
   (i) focal length of the mirror
   (ii) shift of the object

Solution.

(i) $m=\frac{v}{f}-1$

$v_1=(m_1+1)f$.............(1)

$v_2=(m_2+1)f$.............(2)

For $m_2>m_1, v_2>v_1$ then

$v_2-v_1=30$................(3)

$f=\frac{v_2-v_1}{m_2-m_1}$

$f=\frac{30}{5-3}=15cm$

(ii) For $m_2>m_1, u_1>u_2$ then

$\frac{1}{m}=\frac{u}{f}-1$

$u_1=\frac{f}{m_1}+1$.............(1)

$u_2=\frac{f}{m_2}+1$.............(1)

$d=u_1-u_2$

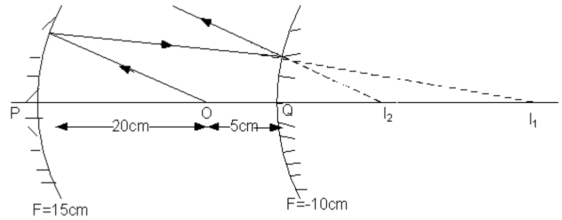
$d=\frac{f}{m_1}-\frac{f}{m_2}$

$d=\frac{15}{3}-\frac{15}{5}$

$\boldsymbol{d=2cm}$

3. A concave mirror **P** of focal length 15cm faces a convex mirror **Q** of focal length 10cm placed 25cm from it. An object is placed between **P** and **Q** at a point 20cm from **P**.
   (i) Determine the distance from **Q** of the image formed by reflection**,** first in **P** and then in **Q**
   (ii) Find the magnification of the image formed in **(i)** above

Solution



Consider the action of a concave mirror

$u = 20cm$, and $f = 15cm$

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$\frac{1}{v} = \frac{1}{15} - \frac{1}{20}$$

$$v = 60cm$$

∴ The image distance from a concave mirror $= 60cm$

Thus, the image distance behind a convex mirror = $60 - (5 + 20)cm = 35cm.$

Consider the action of a convex mirror

The image formed by a concave mirror acts as a virtual object for the convex mirror. Thus

$u = -35cm$ and $f = -10cm$

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$\frac{1}{v} = \frac{1}{-10} - \frac{1}{-35}$$

$$v = -14cm$$

∴ A final virtual image is **14cm** behind the convex mirror

(ii) magnification $m = m_1 m_2$

$$m = \frac{v_1}{u_1} x \frac{v_2}{u_2}$$

$$m = \frac{60}{20} x \frac{14}{35} = 1.2$$

4. A small convex mirror is placed **60cm** from the pole and on the axis of a large concave mirror of radius of curvature **200cm.** The position of the convex mirror is such that a real image of a distant object is formed in the plane of a hole drilled through the concave mirror at its pole.

(a) (i) Draw a ray diagram to show how a convex mirror forms an image of a non-axial point of a distant object

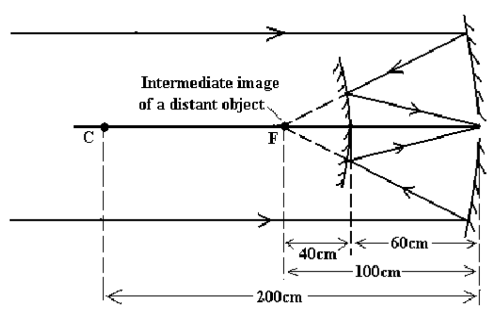
(ii) Suggest a practical application for the arrangement of the mirrors in **a (i)** above**.**

(b) Calculate the

(i) radius of curvature of the convex mirror.

(ii) height of the real image if the distant object subtends an angle of **0.5°** at the pole of the convex mirror.

Solution(b) i)



(ii) The mirror arrangement finds application in a reflecting telescope, a device for viewing distant objects

(b) (i) Consider the action of a concave mirror

The image of a distant object is formed at the principal focus of the concave mirror. This image acts as a virtual object for a convex mirror.

Consider the action of a convex mirror

$u = -40cm$ and $v = 60cm$

$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$ where $r = 2f$

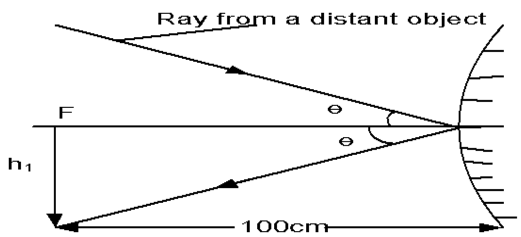
$$\frac{2}{r} = \frac{1}{v} + \frac{1}{u}$$

$$r = \frac{2uv}{u+v} = \frac{2x - 40x60}{-40+60} = -240cm$$

The required radius of curvature r = 240cm

(ii) Consider the magnification produced by a convex mirror

Let h_1 = height of the intermediate image formed by a concave mirror as shown.



$$\tan 0.5° = \frac{h_1}{100}$$

$$h_1 = 0.8727cm$$

h_1 =height of image formed by a convex mirror

$$m = \frac{h_2}{h_1} = \frac{v}{u}$$

$$h_2 = \frac{60}{40}x0.727 = 1.3cm$$

Required image height = 1·3cm

EXERCISE:2

1. Define the terms centre of curvature, radius of curvature, principal focus and focal length of a converging mirror.
2. Distinguish between real and virtual images.
3. Explain with the aid of a concave mirror the term **a caustic surface.**
4. Explain why a parabolic mirror is used in searchlights instead of a concave mirror
5. An object is placed a distance **u** from a concave mirror. The mirror forms an image of the object at a distance **v.** Draw a ray diagram to show the path of light when the image formed is:
   (i) real
   (ii) virtual
6. Give two instances in each case where concave mirrors and convex mirrors are useful.
7. **(i)** Explain the suitability of a concave mirror as a shaving mirror.
   (ii) Explain with the aid of a ray diagram why a convex mirror is used as a car driving mirror.
8. Show with the aid of a ray diagram, that the radius of curvature of a concave mirror is twice the focal length of the mirror
9. Use a geometrical ray diagram for a concave to derive the relation $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$
10. Derive the relation connecting the radius of curvature **r** object distance **u** and image distance **v** of a diverging mirror.
11. An object is placed perpendicular to the principal axis of a concave mirror of focal length f at a distance $(f + x)$ and a real image of the object is formed at a distance $(f + y)$. Show that the radius of curvature **r** of the mirror is given by $r = 2\sqrt{xy}$
12. (i) Define the term **linear magnification.**
    (ii) Show that in a concave mirror, **linear magnification**= $\frac{image\ distance}{object\ distance}$.
    (iii) A concave mirror of focal length 15cm forms an erect image that is three times the size of the object. Determine the object and its corresponding image position.
    (iv) A concave mirror of focal length 10cm forms an image five times the height of its object. Find the possible object and corresponding image positions.

[**Ans: (iii)** $u = 10cm, v = -30cm$, **(iv)** $u = 12cm, v = 60cm$ **OR** $u = 8cm, v = -40cm$]

13. A concave mirror forms on a screen a real image which is twice the size of the object. The object and screen are then moved until the image is five times the size of the object. If the shift of the screen is 30cm, determine the
    (i) focal length of the mirror
    (ii) shift of the object

[**Answers: (i)** $f = 10cm$ **(ii)** $3cm$]

14. A concave mirror of radius of curvature **20cm** faces a convex mirror of radius of curvature **10cm** and is **28cm** from it. If an object is placed midway between the mirrors, find the nature and position of the image formed by reflection first at the concave mirror and then at the convex mirror.
    [**Answer:** A final virtual image is **17·5cm** behind the convex mirror]

15. A small convex mirror is placed **100cm** from the pole and on the axis of a large concave mirror of radius of curvature **320cm.** The position of the convex mirror is such that a real image of a distant object is formed in the plane of a hole drilled through the concave mirror at its pole.

(a) (i) Draw a ray diagram to show how a convex mirror forms an image of a non-axial point of a distant object

(ii) Suggest a practical application for the arrangement of mirrors in **a (i)** above**.**

(iii) Calculate the radius of curvature of the convex mirror

(b) If the distant object subtends an angle of 3×10^{-3}**radians** at the pole of the concave mirror, calculate the

(i) size of the real image that would have been formed at the focus of the concave mirror.

(ii) size of the image formed by the convex mirror

[**Ans: (a) (iii) 150cm (b) (i) 0·48cm (ii) 0·8cm**]

16. A converging mirror produces an image whose length is 2.5 times that of the object. If the mirror is moved through a distance of 5cm towards the object, the image formed is 5 times as long as the object. Calculate the focal length of the mirror. **An(f=25cm)**

17. A concave mirror forms an image half the size of the object. The object is then moved towards the mirror until the image size is three quarters that of the object. If the image is moved by a distance of 0.8cm. find the;

(i) Focal length of the mirror

(ii) New position of the object **An(f=3.2cm, 7.47cm)**

18. A real image is formed 40 cm from a spherical mirror, the image being twice the size of the object. What kind of mirror is it and what is the radius of the curvature.

19. An object is 4cm high. It s desired to form a real image 2cm high and 96cm from the object. Determine the type of mirror required and focal length of the mirror.

20. A dentist holds a concave mirror of focal length 4cm at a distance of 1.5 cm from the tooth. Find the position and magnification of the image which will be formed.

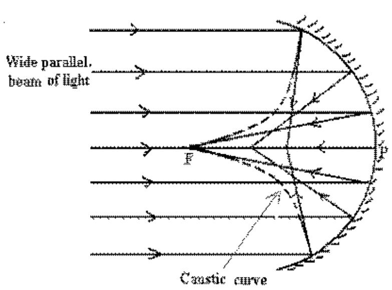
21. A concave mirror of radius of curvature 25cm faces a convex mirror of radius of curvature 20cm with the convex mirror 30cm from a concave mirror. If the object is placed mid way between two mirrors, find the nature and position of the image formed by reflection

(i) By concave mirror

(ii) By convex mirror

REFLECTION OF A PARALLEL WIDE BEAM OF LIGHT AT CURVED MIRRORS

Consider the reflection of a wide parallel beam of light incident on a concave mirror as shown.



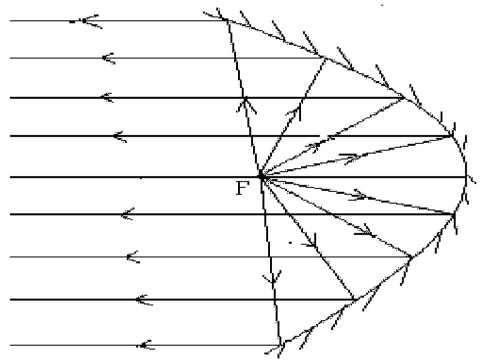
When a wide parallel beam of light is incident on a concave mirror, the different reflected rays are converged to different points. However these reflected rays appear to touch a surface known as a **caustic surface(**A surface on which every reflected ray from the mirror forms a tangent **to)** and has an apex at the principal focus **F**.

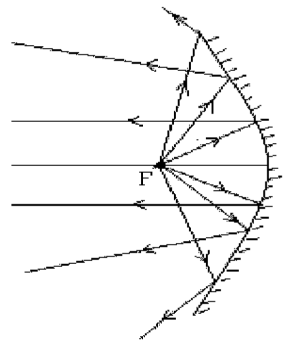
NOTE

(i) The marginal rays furthest from the principal axis are converged nearer to the pole of the mirror than the paraxial rays.

(ii) Similarly, if a wide parallel beam of light is incident on a convex mirror, the different reflected rays appear to have diverged from different points.

COMPARISON OF CONCAVE AND PARABOLIC MIRRORS (Reversing light)





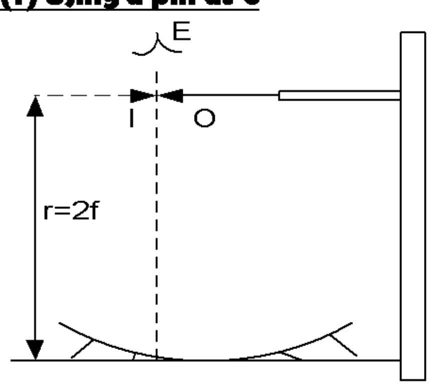
- When a lamp is placed at the principal focus of a concave mirror, only rays from this lamp that strike the mirror at points close to the principle axis will be reflected parallel to the principle axis and those striking at points well away from the principal axis will be reflected in different directions and not as a parallel beam as seen in (b) above. In this case the intensity of the reflected beam practically diminishes as the distance from the mirror increases.
- When a lamp is placed at the principal focus of a parabolic mirror, all rays from this lamp that strike the mirror at points close to and far from the principle axis will be reflected parallel to the principle axis as seen in (a) above. In this case the intensity of the reflected beam remains practically undiminished as the distance from the mirror increases. This accounts for the use of parabolic mirrors as search lights other than concave mirrors.

Note:
A parabolic mirror has the advantage of reflecting the light source placed at the focus parallel to the principal axis with undiminished intensity.

Uses of Parabolic mirrors
They are used as reflectors in search light torches

DETERMINATION OF THE FOCAL LENGTH OF A CONCAVE MIRROR.

Method (1) Using a pin at C



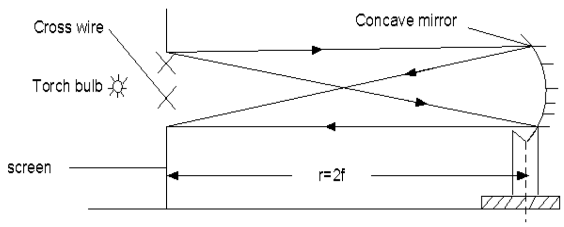
- Place a concave mirror on horizontal bench with its reflecting surface placed upwards
- A pin is then clamped horizontally on a retort stand such that its pointed end lies along the principal axis of the mirror
- Move the pin vertically until the point is located where the pin coincides with its own image
- Measure the distance r from the mirror to the pin
- The focal length $f = \frac{r}{2}$

NOTE :

(i) In the position where there is no parallax between the object pin and its image, there is no relative motion between the object and its image when the observer moves the head from side to side.

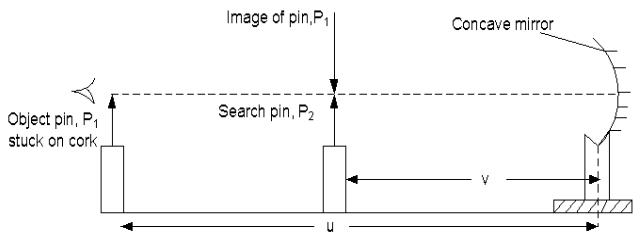
(ii) When the pin coincides with its image, the rays are incident normal to the mirror and are thus reflected along their own path. Therefore the pin coincides with its image at the centre of curvature of the mirror.

Method (2) Using an illuminated object at C



- An object in this, consists of a hole cut in a white screen made of cross-wire illuminated from behind by a source of light.
- A concave mirror is mounted in a holder, and moved to and from in front of the screen until a sharp image of the cross-wire is formed on the screen adjacent to the cross-wire.
- When this has been done, both the image and the object are at the same distance from the mirror, and hence both must be situated in a plane passing through the centre of curvature and at right angles to the axis.
- The distance between the mirror and the screen is measured and this is the radius of curvature, **r**.
- Half of this distance, **r**, is the focal length, f ($f = {}^{r}/_{2}$).

Method (3) Using no parallax method in locating V



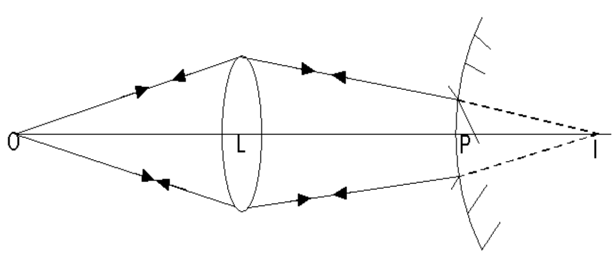
- An object pin **P_1** is placed at a distance **u** in front of a mounted concave mirror so that its tip lies along the principal axis of the mirror and it forms an inverted image.
- The distance of the object pin from the mirror is measured, u.
- A search pin **P_2** placed between the mirror is and pin **p_1** is adjusted until it coincides with the image of pin **p_1** by no-parallax method.
- The distance **v** of pin **p_2** from the mirror is measured.
- The procedure is repeated for several values of **u** and the results are tabulated including values of uv, and $u + v$.
- A graph of uv against $u + v$ is plotted and the slope **s** determined
- The focal length **f** of the mirror if $f = s$.

NOTE:

If a gaph of $\frac{1}{U} agaisnt\ \frac{1}{V}$ is plotted, then each intercept C of such a graph is equal to $\frac{1}{f}$. Hence $f = \frac{1}{C}$

AN EXPERIMENT TO DETERMINE THE FOCAL LENGTH OF THE CONVEX MIRROR

Method (1) Using a convex lens.



- Place an object O in front of convex lens and locate the position of real image
- Measure the distance LI and record
- The convex mirror is placed between the lens and image I with its reflecting surface facing the lens. Move the mirror along OI until the image I coincides with object
- Measure the distances PI and LP
- Focal length f is obtained from $f = \frac{LI - LP}{2}$

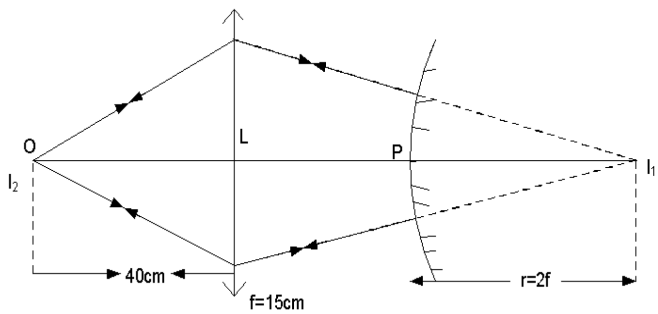
NOTE ;

When the incident rays from an object are reflected back along the incident path, a real inverted image is formed besides the object in which case the rays strike the mirror normally. Therefore they will if produced pass through the centre of curvature of the mirror thus distance **PI = radius of curvature**

Examples

1. An object **O** is placed **40cm** in front of a convex lens of focal length **15cm** forming an image on the screen. A convex mirror situated **4cm** from the lens in the region between the lens and the screen forms the final image besides object **O**.
   (i) Draw a ray diagram to show how the final image is formed.
   (ii) Determine the focal length of the convex mirror.

Solution



Consider the action of a convex lens
$u = 40cm$, and $f = 15cm$

From $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$

$$\frac{1}{15} = \frac{1}{v} + \frac{1}{40}$$

$$v = 24cm$$

The radius of curvature $r = (24 - 4)cm$
$= 20cm$

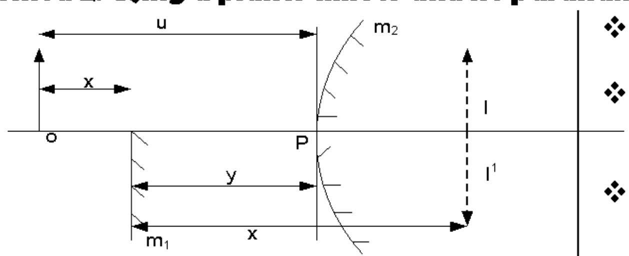
Using the relation $r = 2f$

$\Rightarrow 2f = 20cm$

$\therefore f = 10cm$

Thus $f = -10cm$ "The centre of curvature of a convex mirror is virtual"

Method 2: Using a planer mirror and no parallax method



- An object pin O is placed in front of convex mirror m_2 such that it forms a virtual diminished image at I
- The distance u of object O from convex mirror is measured
- A plane mirror m_1 is then placed between object O and the convex mirror such that it covers half aperture of convex mirror
- The plane Mirror **M** is adjusted until its own image of **O** coincides with **I** by no parallax method.
- Measure the distance x and y
- F can be calculated from $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

Where $u = (x + y)$ and $v = -(x - y)$

Note:

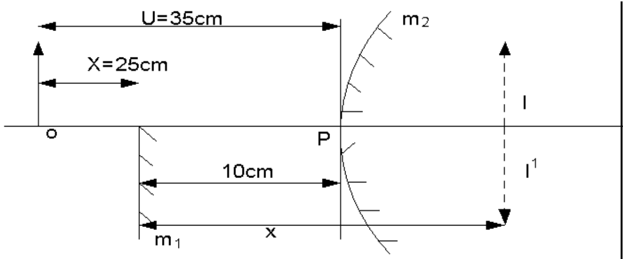
(i) The two images coincides when they are as far behind the plane mirror as the object is in front.

(ii) Substituting for $u = (x + y)$and $v = -(x - y)$ in the mirror formula gives $f = \frac{y^2 - x^2}{2y}$

Examples

1. A plane mirror is placed 10cm infront of a convex mirror so that it covers half of the mirror surface. Apin 25cm infront of a plane mirror gives an image which coincide with that of the pin in the convex mirror. Find the focal length of the convex mirror.

Solution



$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

Where $u = (x + y)$ and $v = -(x - y)$

$$\frac{1}{f} = \frac{1}{35} + \frac{1}{-(25-10)}$$

$$\frac{1}{f} = \frac{1}{35} - \frac{1}{15}$$

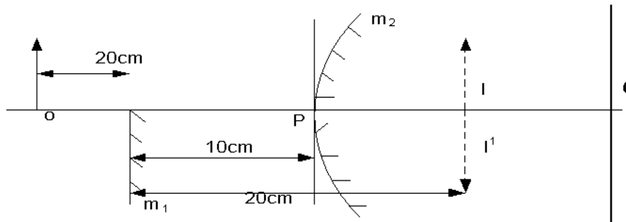
$$\frac{1}{f} = \frac{-20}{525}$$

$$f = \frac{525}{-20}$$

$$f = -26.25cm$$

2. A plane mirror is placed **10cm** in front of a convex mirror so that it covers about half of the mirror surface. A pin **20cm** in front of the plane mirror gives an image in it, which coincides with that of the pin in the convex mirror. Find the focal length of the convex mirror.

Solution



Consider the action of a convex mirror

$u = 30cm$ and $v = -(20 - 10) = -10cm$

"The image formed is virtual "

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{f} = \frac{1}{30} + \frac{1}{-10}$$

$$f = -15cm$$

EXERCISE:3

1. Describe an experiment to determine the focal length of a concave mirror.
2. You are provided with the following pieces of apparatus: A screen with cross wires, a lamp, a concave mirror, and a meter ruler. Describe an experiment to determine the focal length of a concave mirror using the above apparatus.
3. Describe an experiment, including a graphical analysis of the results to determine the focal length of a concave mirror using a no parallax method.
4. Describe an experiment to measure the focal length of a convex mirror
5. Describe how the focal length of a diverging mirror can be determined using a convex lens.
6. Describe how the focal length of a convex mirror can be obtained using a plane mirror and the no parallax method.
7. A plane mirror is placed at a distance d in front of a convex mirror of focal length f such that it covers about half of the mirror surface. A pin placed at a distance L in front of the plane mirror gives an image in it, which coincides with that of the pin in the convex mirror. With the aid of an illustration, Show that $2df = d^2 - L^2$

REFRACTION OF LIGHT

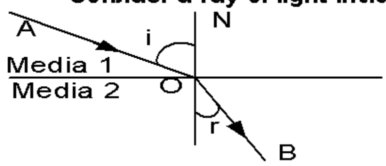
Refraction is the change of direction of light propagation of light as it travels from one medium to another.

Explanation of refraction

The bending of light is as a result of the change in speed as light travels from one medium to another. The change in speed of light usually leads to the change in direction unless if the ray is incident normally. The speed of light in air is higher than the speed of light in glass or water.
Glass and water are therefore said to be denser than air also is denser than water.

LAWS OF REFRACTION

Consider a ray of light incident on an interface between two media as shown.



O = Point of incidence.

OA = Incident ray
OB = Refracted ray.
ON = Normal at O
$\angle i$ =Angle of incidence
$\angle r$ =Angle of refraction

LAW 1: The incident ray, the refracted ray and the normal at the point of incidence all lie in the same plane.

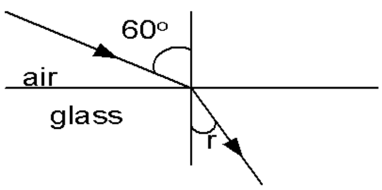
LAW 2: The ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant for a given pair of media.

This is called **Snell's law.**
This constant ratio is called Refractive index, (*n*).

$n = \frac{\sin i}{\sin r}$ where;
i = angle of incidence r = angle of refraction.

Examples

1.



Find the angle of refraction if the refractive index of glass is 1.52

Solution

$n = \frac{\sin i}{\sin r}$

$1.52 = \frac{\sin 60}{\sin r}$

Sin r = $\frac{0.8666}{1.52}$

Sin r = 0.569
r = $sin^{-1}(0.569)$
r = 34.7°

2. The angle of incidence of water of refractive index 1.33 is $45°$. Find the angle of refraction.

Solution

$n = \frac{\sin i}{\sin r}$

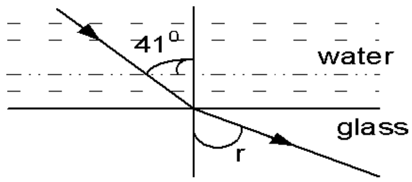
$1.52 = \frac{\sin 45}{\sin r}$

Sin r = $\frac{0.707}{1.33}$

Sin r = 0.531
r = $sin^{-1}(0.531)$
r = 34.7°

Exercises4

1. The angle of incidence is 30° and angle of refraction is 19°. Find the refractive index of the material
2. A ray of light is incident in air at an angle of 30°. Find the value of angle of refraction, r, if the refractive index is 1.5.
3. A ray of light is incident on a water- glass boundary at an angle of 41° as shown below.



Calculate the angle of refraction, if the refractive indices of water and glass are 1.33 and 1.50 respectively

Refractive index n

Refractive index of a material is the ratio of the sine of angle of incidence to the sine of angle of refraction for a ray of light traveling from a vacuum to a given medium.

OR

Is the ratio of the speed of light in a vacuum to speed of light in a medium.

Thus **Refractive index**, $n = \frac{speed\ of\ light\ in\ a\ vacuum\ (c)}{speed\ of\ light\ in\ a\ medium\ (v)}$

Where speed of light in a vacuum $c = 3.0\ x\ 10^8\ ms^{-1}$.

NOTE:

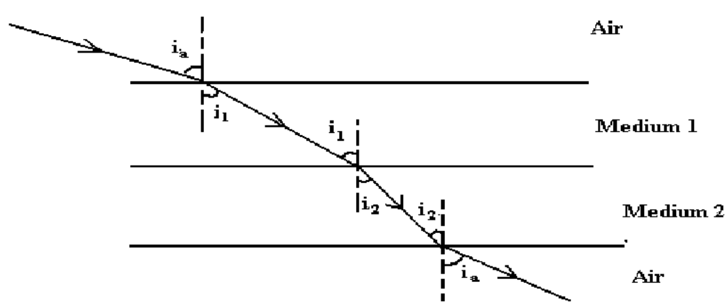
The refractive index, **n** for a vacuum is **1**. However if light travels from air to another medium, the value of **n** is slightly greater than **1**. For example, $n = 1{\cdot}33$ for water and $n = 1{\cdot}5$ for glass.

THE PRINCIPLE OF REVERSIBILITY OF LIGHT.

It states that the paths of light rays are reversible.
This means that a ray of light can travel from medium **1** to **2** and from **2** to **1** along the same path.

GENERAL RELATION BETWEEN n AND sin i

Consider a ray of light moving from air through a series of media **1, 2** and then finally emerge into air as shown.



At **air – medium 1** interface, Snell's gives $\frac{\sin i_a}{\sin i_1} = n_1$

$\Rightarrow \quad \sin i_a = n_1 \sin i_1$-- (i)

At **air - medium 2** interface, Snell's gives $\frac{\sin i_a}{\sin i_2} = n_2$

$\Rightarrow \quad \sin i_a = n_2 \sin i_2$---------------------------------------**(ii)**

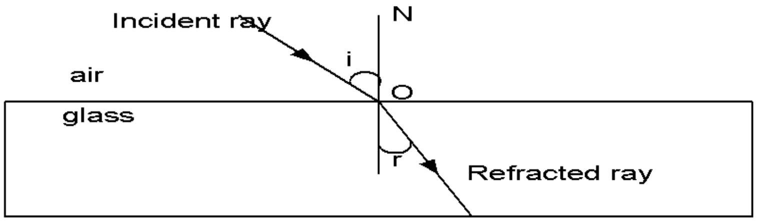
Equating equation (i) and (ii) gives

$n_1 \sin i_1 = n_2 \sin i_2$

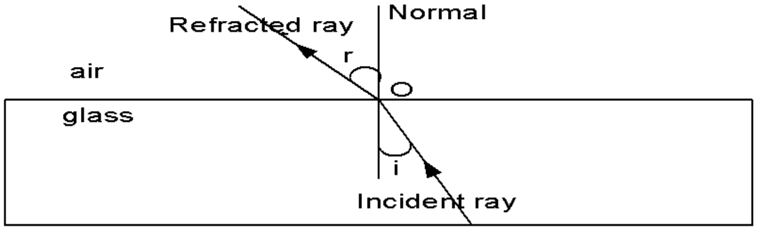
$\therefore$ $\boxed{n \sin i = a\ constant.}$

N.B:

1. For a ray travelling from a less dense medium to a denser medium e.g from air to glass, it is refracted towards the normal since there is a decrease in speed of light.



2. If the ray travels from a denser to a less dense medium e.g. from glass to air, it will be reflected away from the normal just because there will be an increase in the speed of light.

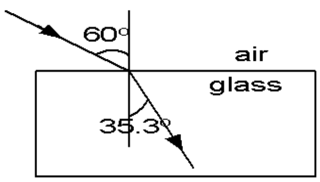


3. When the incident ray meets a refracting surface at 90°, it is not refracted at all
4. The speed of light reduces when it travels into a dense medium

Examples:

1. Calculate the refractive index of the glass if a ray of light is incident on it at an angle of $60°$ and it is refracted at an angle of $35.3°$.

Solution:



$n \sin i = constant$

$n_a sini_a = n_g sini_g$

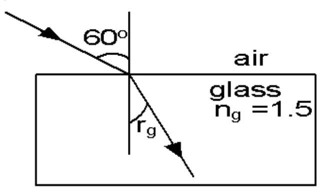
$1 \text{ x } sin60 = n_g \text{x } \sin 35.3$

$\frac{0.866}{0.5778} = \frac{0.5778}{0.5778} ng$

∴ The refractive index of glass is 1.5

2. A ray of light in air makes an angle of incidence of $60°$ with the normal to glass surface of refractive index 1.5. What is the angle of refraction?

Solution



$n_a sini_a = n_g sini_g$

$1 \text{ x } sin60 = 1.5 sinr_g$

$\frac{0.866}{1.5} = \frac{1.5}{1.5} sinrg$

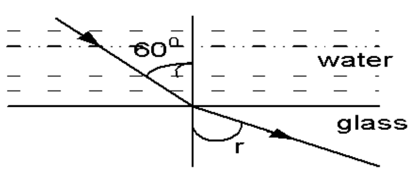
$sinr_g = 0.5773$

$r_g = sin^{-1} 0.5773$

$r_g = 35.3°$

The angle of refraction is 35.3°

3. A monochromatic beam of light is incident at **60°** on a water-glass interface of refractive index $1{\cdot}33$ and $1{\cdot}5$ respectively as shown



Calculate the angle of reflection **r.**

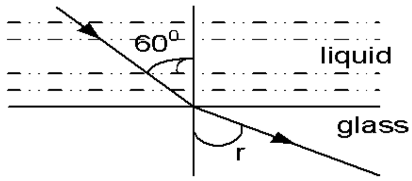
Solution:

Applying Snell's law gives $n_w sini_w = n_g sini_g$

$1.33 sin60 = 1.5 sinr$

Thus $\angle r = 50{\cdot}2°$

4. A monochromatic ray of light is incident from a liquid on to the upper surface of a transparent glass block as shown.



Given that the speed of light in the liquid and glass is $2.4 \times 10^8\ ms^{-1}$ and $1.92 \times 10^8\ ms^{-1}$ respectively, find the angle of refraction, **r.**

Solution:

$$n_l sin i_l = n_g sin i_g$$

$$\frac{c}{v_l} sin60 = \frac{c}{v_g} sinr$$

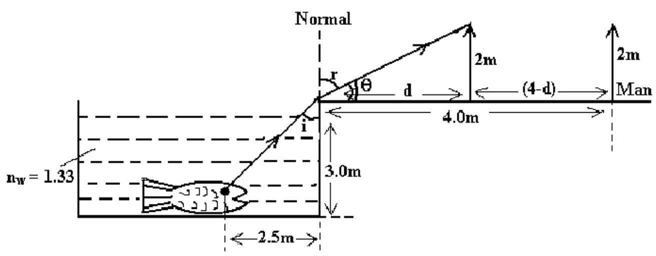
$$sinr = \frac{v_g}{v_l} sin60$$

$$sinr = \frac{1.92 \times 10^8}{2.4 \times 10^8} sin60$$

$$\Rightarrow \angle r = 43.9°.$$

5. A small fish is 3·0m below the surface of the pond and 2·5m from the bank. A man 2·0m tall stands 4·0m from the pond. Assuming that the sides of the pond are vertical, calculate the distance the man should move towards the edge of the pond before movement becomes visible to the fish. **(Refractive index of water = 1·33).**

Solution



From the diagram, $tan\ i = \frac{2.5}{3}$

$\Rightarrow \angle i = 39.81°$

Applying Snell's law at the edge of the pond gives

$$n_w sin i_w = n_g sin i_g$$

$$1{\cdot}33\ sin\ 39{\cdot}81° = 1\ sin\ r$$

$$\Rightarrow \angle r = 58.4°$$

Thus $\angle\theta = 90° - 58.4° = 31.6°$

From the diagram $tan\ \theta = \frac{2}{d}$

$$\Rightarrow d = \frac{2}{\tan 31.6}$$

$$\therefore d = 3{\cdot}2m.$$

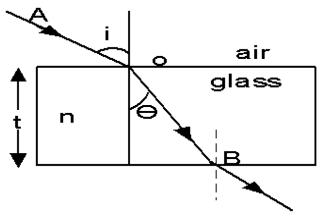
Thus required distance traveled $= 4 - d$

$= 4 - 3{\cdot}2$

$= 0{\cdot}8m$

6. A monochromatic light incident on a block of material placed in a vacuum is refracted through an angle of θ. If the block has a refractive index **n** and is of thickness **t**, show that light takes time of $\frac{n\,t\,sec\theta}{c}$ to emerge

Solution



Distance travelled in the block is OB

$$cos\theta = \frac{t}{OB}$$

$$OB = \frac{t}{cos\theta} = t\ sec\theta$$

$$speed = \frac{distance}{time}$$

$$time = \frac{OB}{V} = \frac{t\ sec\theta}{V}$$

$$n = \frac{speed\ of\ light\ in\ a\ vacuum}{speed\ of\ light\ in\ a\ medium} = \frac{C}{V}$$

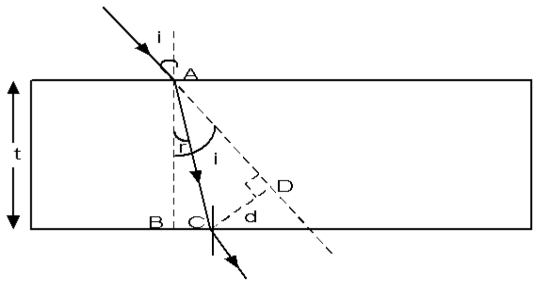
$$V = \frac{C}{n}$$

$$time = \frac{t\ sec\theta}{V} = \frac{t\ sec\theta}{\left(\frac{C}{n}\right)} = \frac{\boldsymbol{n\,t\,sec\theta}}{\boldsymbol{c}}$$

SIDE WISE DISPLACEMENT OF LIGHT RAYS.

When light travels from one medium to another, its direction is displaced side ways. This is called lateral displacement.

Consider a ray of light incident at an angle **i** on the upper surface of a glass block of thickness **t**, and then suddenly refracted through an angle **r** causing it to suffer a sidewise displacement **d**.



From Δ ABC, $AC = \frac{t}{\cos r}$ -----------------.(i)

From the diagram, $\angle CAD = (i - r)$

From Δ ACD, $AC = \frac{d}{\sin(i-r)}$ ----------(ii)

Equating equation (i) and (ii) gives

$$\frac{t}{\cos r} = \frac{d}{\sin(i-r)}$$

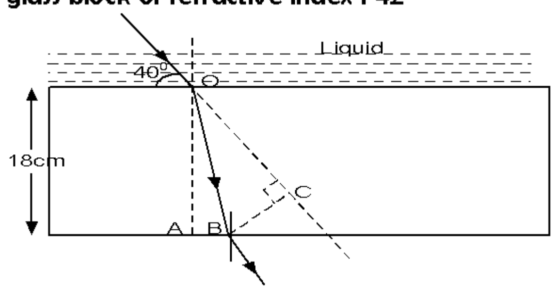
$$\Rightarrow \quad \boldsymbol{d} = \frac{t\sin(i-r)}{\cos r}$$

NOTE :

The horizontal displacement of the incident ray , $BC = t.\tan r$

Examples:

1. The figure below shows a monochromatic ray of light incident from a liquid of refractive index 1·33 onto the upper surface of a glass block of refractive index 1·42

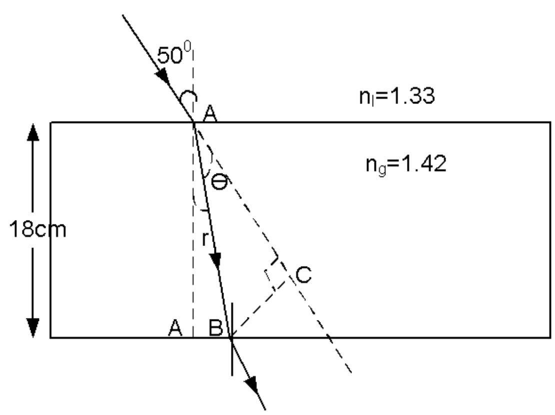


Calculate the;

(i) horizontal displacement AB.

(ii) lateral displacement BC of the emergent light.

Solution



(i) Applying Snell's law at the liquid- glass interface gives,

$$n_l \sin i_l = n_g \sin i_g$$

$$1{\cdot}33 \sin 50^\circ = 1{\cdot}42 \sin r$$

$$\Rightarrow \quad \angle r = 45.8^\circ$$

Horizontal displacement $AB = t\tan r$

$= 18\tan 45{\cdot}8^\circ$

$= \mathbf{18.51cm}$

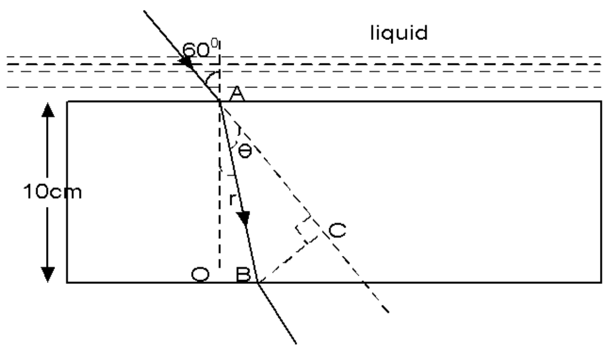
(ii) Lateral displacement $d = \frac{t\sin(i-r)}{\cos r}$

$$\boldsymbol{d} = \frac{18\sin(50-45.8)}{\cos 45.8}$$

$$\boldsymbol{d} = \mathbf{1.89cm.}$$

2. Monochromatic light is incident from the liquid on the upper surface of transparent glass block where the sides of block are plane and parallel. If the speed of light in the liquid $2.42x10^8 ms^{-1}$ and speed of light in glass sis $1.92x10^8 ms^{-1}$. Calculate the lateral displacement of the emergent beam

Solution



$$n = \frac{speed\ of\ light\ in\ a\ vacuum}{speed\ of\ light\ in\ a\ medium} = \frac{C}{V}$$

$$n_L = \frac{3.0x10^8}{2.42x10^8} = 1.239$$

$$n_G = \frac{3.0x10^8}{1.92x10^8} = 1.563$$

$n\ sin\ i = constant$

$1.239xsin60 = 1.563sinr$

$r_R = 34.1°$

$1\ x\ sin60 = 1.596sinr_B$

$r = 43.4°$

$\theta = 60 - 43.4 = 16.6°$

$$Cos43.4 = \frac{10}{AB}$$

$AB = 13.76cm$

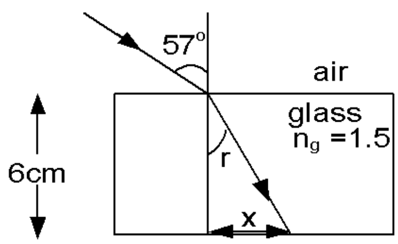
$$Sin16.6 = \frac{BC}{13.76}$$

$BC = 3.93cm$

Further examples

1. **Calculate the horizontal displacement of ray of light incident at an angle of 57° on a glass block 6cm thick whose refractive index is 1.5**

Solution



$n_a sini_a = n_g sini_g$

$1\ x\ sin57 = 1.5sinr_g$

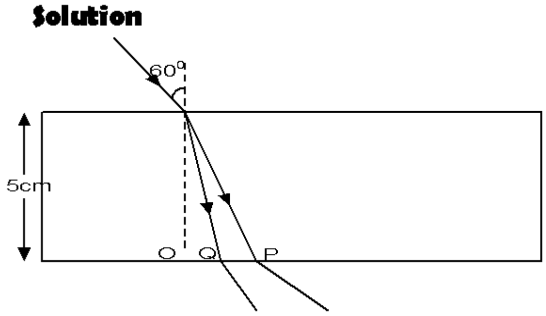
$r_g = 35.3°$

$$\tan 34 = \frac{x}{6}$$

$x = 4.04cm$

2. **Light consisting of blue and red light is incident in air glass interface. The two colours emerge from glass block at two points P and Q respectively. If the speed of blue and red light in glass are $1.88x10^8 ms^{-1}$ and $1.94x10^8 ms^{-1}$ respectively. Calculate the distance PQ**

Solution



$$n = \frac{speed\ of\ light\ in\ a\ vacuum}{speed\ of\ light\ in\ a\ medium} = \frac{C}{V}$$

$$n_B = \frac{3.0x10^8}{1.88x10^8} = 1.596$$

$$n_R = \frac{3.0x10^8}{1.94x10^8} = 1.546$$

$n\ sin\ i = constant$

$1\ x\ sin60 = 1.546sinr_R$

$r_R = 34.1°$

$1\ x\ sin60 = 1.596sinr_B$

$r_B = 32.9°$

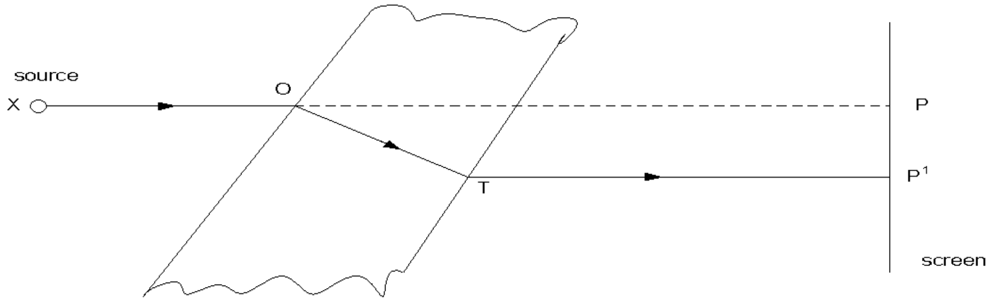
$$tanr_R = \frac{5}{OQ}$$

$OQ = 5\ tan34.1° = 3.38cm$

$OQ = 5\ tan32.9° = 3.23cm$

$PQ = 3.28 - 3.23 = 0.15cm$

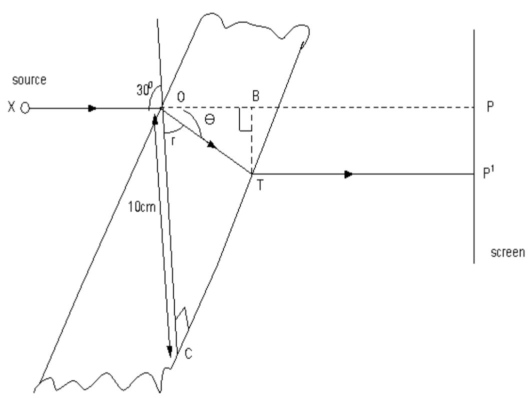
3. **A monochromatic source in air sends narrow beam of light perpendicular to a screen $2.0m$ away. The beam strikes the screen at P. A glass block of refractive index 1.5 and thickness 10cm is inserted as shown below so that the beam strikes it at an angle of 30°**



Find;

- (i) **The angle of refraction at the first surface**
- (ii) **The distance OT**
- (iii) **The speed of the beam through glass block**
- (iv) **Time taken to cover distance OT**
- (v) **Lateral displacement of the beam**

Solution



(i) $n \sin i = constant$

$1x\sin 30 = 1.5\sin r$

$r = 19.47°$

(ii) $\Delta OCT; \cos 19.47 = \frac{0.1}{OT}$

$OT = 0.11m$

(iii) $n = \frac{speed\ of\ light\ in\ a\ vacuum}{speed\ of\ light\ in\ a\ medium} = \frac{C}{V}$

$V_g = \frac{3.0x10^8}{1.5} = 2.0x10^8 ms^{-1}$

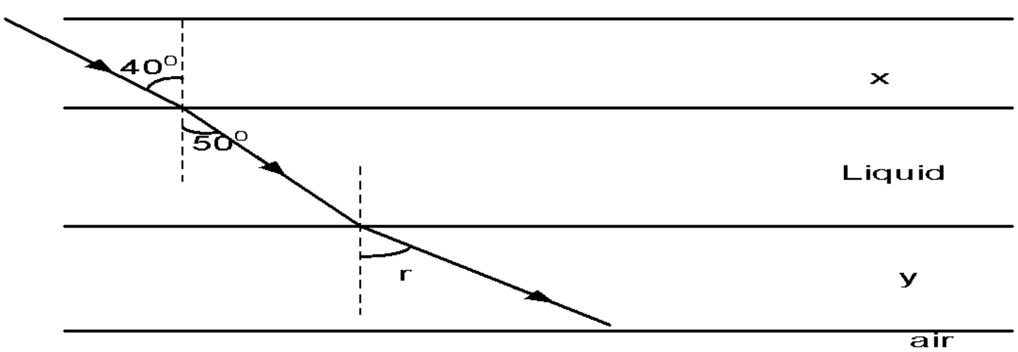
(iv) $time = \frac{distance}{speed} = \frac{0.11}{2.0x10^8} = 5.5x10^{-10}s$

(v) $\theta = 30 - 19.47 = 10.53°$

$\sin 10.53 = \frac{BT}{0.11}$

$BT = 0.02m$

4. **The figure below shows a layer of liquid confined between two transparent plates X and Y of refractive index of 1.54 and 1.44 respectively. A ray of monochromatic light making an angle of $40°$ with normal to the interface between medium X and liquid is refracted through an angle of $50°$ by the liquid. Find**



- **(i)** **Refractive index of the liquid**
- **(ii)** **The angle of refraction in medium y**
- **(iii)** **Minimum angle of incidence x for which the ray light will not emerge from the medium y**

Solution

(i) $n \sin i = constant$

glass x- liquid interface

$1.54\sin 40 = n_l \sin 50$

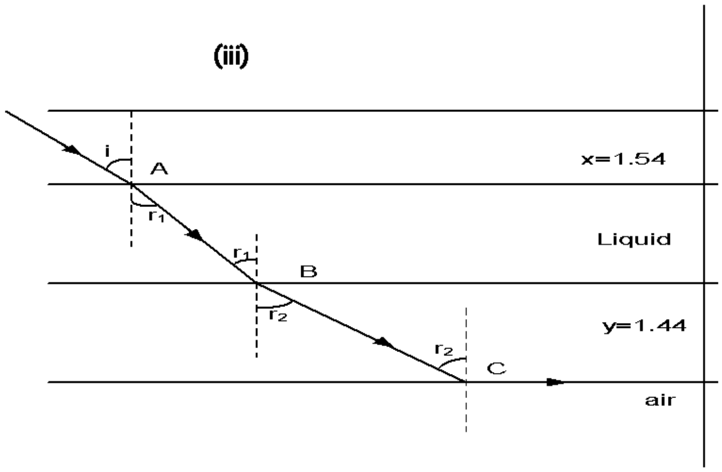
$n_l = 1.29$

(ii) **liquid – glass y interface**

$1.29\sin 50 = 1.44\sin r$

$r = 43.3°$

(iii)



At C: $1xsin90 = 1.44x\sin r_2$

$r_2 = 44.0°$

At B: $1.44xsin44 = 1.29x\sin r_1$

$r_1 = 50.8°$

At A: $1.29xsin50.8 = 1.54x\sin i$

$i = 40.5°$

Exercise:5

(1) What is meant by **refraction of light?**

(2) (i) State the laws of refraction of light.

(ii) State what brings about refraction of light as it travels from one medium to another.

3. A beam of light is incident on a surface of water at an angle of 30° with the normal to the surface. The angle of refraction in water is 22°. Find the speed of light in water if it is 3 x 10^8 ms^{-1}.

4. A ray of light in air makes an angle of 30° with the surface of a rectangular glass block of refractive index 1.5. What is the angle of refraction?. **An**$(19.5°)$

5. A ray of light travelling from a liquid to air has an angle of incidence of $40°$ and an angle of refraction of $60°$. Find the refractive index of the liquid. **An**(1.35)

6. **(i)** What is meant by the **refractive index** of a material?

(ii) Light of two colours blue and red is incident at an angle γ from air to a glass block of thickness **t**. When blue and red lights are refracted through angles of θ_b and θ_r respectively, their corresponding speeds in the glass block are V_b and V_r**.** Show that the separation of the two colours at the bottom of the glass block $d = \frac{t}{C}\left(\frac{V_r}{cos\theta_r} - \frac{V_b}{cos\theta_b}\right)sin\gamma$. Where $\theta_r > \theta_b$ and **C** is the speed of light in air.

(iii) Light consisting of blue and red is incident at an angle of **60**° from air to a glass block of thickness **18cm.** If the speeds of blue and red light in the glass block are $1.86 \times 10^8\ ms^{-1}$ and $1.92 \times 10^8\ ms^{-1}$ respectively, find the separation of the two colours at the bottom of the glass block. [**Answer: 0.54cm**]

7. Show that when the ray of light passes through different media separated by plane boundaries, $nsin\phi = constant$ where n is the absolute refractive index of a medium and ϕ is the angle made by the ray with the normal in the medium.

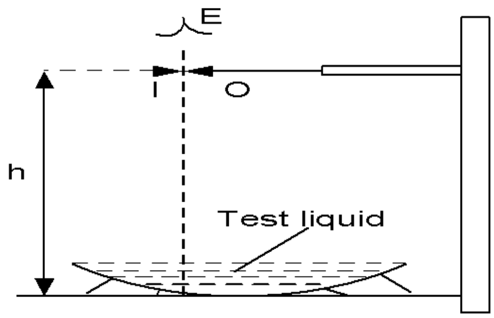
8. Show that when the ray of light passes through different media **1** and **2** separated by plane boundaries, $_1n_2 \times {}_2n_1 = 1$ where **n** is the refractive index of a medium.

9. Show that when the ray of light passes through different media **1,2** and **3** separated by plane boundaries, $_1n_3 = {}_1n_2 \times {}_2n_3$ where **n** is the refractive index of a medium.

10. Show that a ray of light passing through a glass block with parallel sides of thickness **t** suffers a sidewise displacement $d = \frac{t\,sin(\phi-\lambda)}{\cos\lambda}$, where ϕ is the angle of incidence and λ is the angle of refraction.

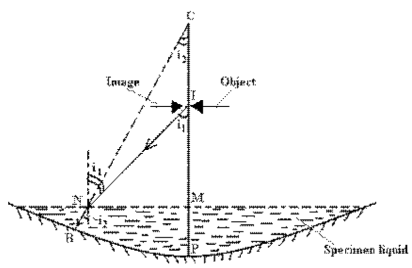
DETERMINATION OF REFRACTIVE INDEX

(i) Concave mirror method (refractive index of small quantity of liquid)



- The concave mirror is placed on a table with its reflecting surface upwards.
- An optical pin is clamped horizontally on a retort stand above the mirror with one end on the principal axis of the mirror.
- The pin is adjusted vertically while viewing from above until when a point is located whee the pin coincides with its image without parallax
- The distance of the pin from the pole of the mirror, R cm is measured and recorded
- The test liquid is poured into the mirror to a depth, dcm
- The pin is adjusted to locate the point of coincidence of the pin and the image .
- The distance of the pin above the liquid hcm is measured and recorded.
- Refractive index of the liquid n is obtained from $n = \frac{R-d}{h}$ or $n = \frac{R}{h}$

Theory



PROOF

For refraction at **N,** $n_a sin i_1 = n_l \sin i_2$------- (i)

From the diagram,

$\sin i_2 = \frac{NM}{NC}$ and $\sin i_1 = \frac{NM}{NI}$

Equation (i) now becomes

$$n_a\left(\frac{NM}{NI}\right) = n_l\left(\frac{NM}{NC}\right)$$

On simplifying, $n_l = \frac{NC}{NI}$

But **N** is very close to **M** hence NC ≈ MC and NI ≈ MN

⇒ $$n_l = \frac{MC}{MI}$$

Also for a small quantity of the liquid, **M** is close to **P** ⇒ MC ≈ CP = **r**, and MI ≈ IP.

Thus $n_l = \frac{r}{IP}$

NOTE :

If the specimen liquid is of reasonable quantity, then its depth **d** can not be ignored. In this case,

$$\boxed{n_l = \frac{MC}{MI} = \frac{r-d}{IP-d}}$$

Examples

1. A liquid is placed in a concave mirror to a depth of **2cm**. An object held above the liquid coincides with its own image when it is **45.5cm** from the pole of the mirror. If the radius of curvature of mirror is **60cm**, calculate the refractive index of the liquid
$$n = \frac{r-2}{x-2} = \frac{60-2}{45.5-2} = 1.33$$
2. A liquid is poured in to a concave mirror to a depth of **2·0cm**. An object held above the liquid coincides with its own image when its **27·0cm** above the liquid surface. If the radius of curvature of the mirror is **40·0cm**, calculate the refractive index of the liquid.
$$n = \frac{r-2}{x} = \frac{40-2}{27} = 1.4$$
3. A small concave mirror of focal length **8cm** lies on a bench and a pin is moved vertically above it .At what point will the pin coincide with its image if the mirror is filled with water of refractive index ${}^4/_3$.

Solution

For a small concave mirror, the quantity of water is small that its depth **d** can be ignored

Using the relation $n = \frac{r}{x}$

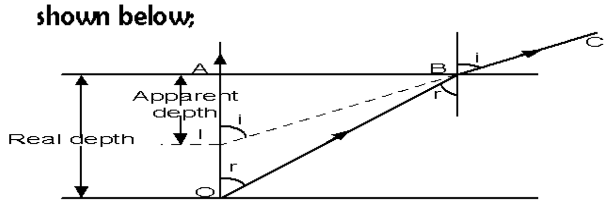
Where $r = 2f = 2 \times 8 = 16cm$

$$x = \frac{16}{{}^4/_3} = 12cm$$

Therefore the pin coincided with its image at a height of **12cm** above the mirror

REAL AND APPARENT DEPTH

When a glass block e.g. is placed on top of the object, the object when viewed appears displaced as shown below;



- A ray OB is refracted into the air along BC. To an observer at C, ray BC appears to come from IB
- O is the apparent position and O is the real position of the object

  Applying sneils law at B

$$n \sin i = constant$$

$$n_g sinr = n_a \text{sini} \ldots\ldots\ldots\ldots\ldots\ldots.(1)$$

Consider ΔIBA; $\sin i = \frac{AB}{IB}$

Consider ΔOBA; $\sin r = \frac{AB}{OB}$

Putting into equation 1; $n_g \frac{AB}{OB} = \frac{AB}{IB}$

$$n_g = \frac{OB}{IB}$$

For small angles $OB \approx OA \ and \ IB \approx AI$

$$n_g = \frac{OA}{IA} = \frac{Real\ depth}{Apparent\ depth} = \frac{t}{t-d}$$

$$\boxed{d = t\left(1 - \frac{1}{n}\right)}$$

Where d is the apparent displacement and t is the real depth.

NOTE

(i) The apparent displacement **d** of an object **O** is independent of the position of **O** below the glass block. Thus the same expression above gives the displacement of an object which is some distance in air below a parallel-sided glass block.

(ii) If there are different layers of different transparent materials resting on top of each other, the apparent position of the object at the bottom can be found by adding the separate displacements due to each layer.

Examples

1. A microscope is focused on a mark on a table. When the mark is covered by a plate of glass 2cm thick, the microscope has to be raised 0.67cm for the mark to be once more in focus.

   Solution

$$n = \frac{t}{t-d}$$

$$n = \frac{2}{2 - 0.67}$$

n = 1.5

2. An object 6cm below the tank of water of refractive index of 1.33. determine the displacement of object to the observer directly above the tank

   Solution

$$d = t\left(1 - \frac{1}{n}\right)$$

$$d = 6\left(1 - \frac{1}{1.33}\right)$$

$$d = 1.48cm$$

3. A tank contains slab of glass 8cm thick and of refractive index 1.52, above this is a liquid 10cm thick of refractive index 1.45 and floating on it is 3cm of water of refractive index 1.33. find the apparent position of the mark below the tank

   Solution

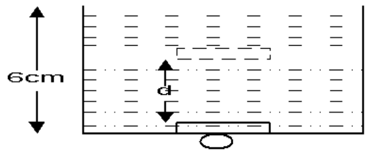
$$d = t\left(1 - \frac{1}{n}\right)$$

$$d = 8\left(1 - \frac{1}{1.52}\right) + 10\left(1 - \frac{1}{1.45}\right) + 3\left(1 - \frac{1}{1.33}\right)$$

$$d = 6.583cm$$

4. An object at a depth of $6{\cdot}0cm$ below the surface of water of refractive index $4/_3$ is observed directly from above the water surface. Calculate the apparent displacement of the object

   Solution



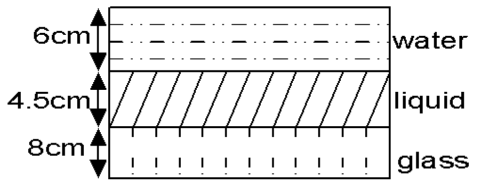
Using the relation $d = t\left(1 - \frac{1}{n}\right)$

$$d = 6\left(1 - \frac{3}{4}\right)$$

$$d = 1.5cm$$

5. A tank contains a slab of glass **8cm** and refractive index **1.6**. Above this is a depth of **4.5cm** of a liquid of refractive index **1.5** and upon this floats **6cm** of water of refractive index $^4/_3$ calculate the apparent displacement of an object at the bottom of the tank to an observer looking down wards directly from above.

Solution



Using the relation $d = t\left(1 - \frac{1}{n}\right)$

Apparent displacement $d = d_w + d_l + d_g$

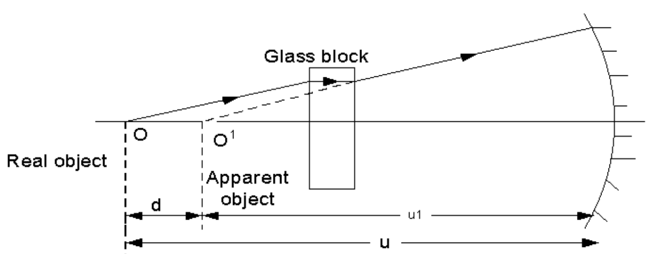
$$6\left(1 - \frac{3}{4}\right) + 4.5\left(1 - \frac{1}{1.5}\right) + 8\left(1 - \frac{31}{1.6}\right)$$

$$\boldsymbol{d} = \mathbf{1.5} + \mathbf{1.5} + \mathbf{3}$$

$$\boldsymbol{d} = \boldsymbol{6cm.}$$

6. A small object is placed **20cm** in front of a concave mirror of focal length **15cm.** A parallel-sided glass block of thickness **6cm** and refractive index **1.5** is then placed between the mirror and the object. Find the shift in the position and size of the image

Solution



Consider the action of a concave mirror in the absence of a glass block

$u = 20cm$ and $f = 15cm$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{15} = \frac{1}{20} + \frac{1}{v}$$

$$v = 60cm$$

Thus in the absence of a glass block**, image distance** $= 60cm$

In this case**, magnification** $m = \frac{v}{u} = \frac{60}{20} = 3$

Consider the action of a glass block

$$d = t\left(1 - \frac{1}{n}\right)$$

$$d = 6\left(1 - \frac{1}{1.5}\right) = 2cm$$

Thus in the presence of a glass block, object distance **u'** $= (20 - 2)\,cm = 18cm$

"The object is displaced and it appears to be 18cm in front of the mirror"

Consider the action of a concave mirror in the presence of a glass block

u' $= 18cm$ and $f = 15cm$

$$\frac{1}{f} = \frac{1}{u^1} + \frac{1}{v^1}$$

$$\frac{1}{15} = \frac{1}{18} + \frac{1}{v^1}$$

$$v^1 = 90cm$$

$\therefore$ **The required shift in the image position**

$= v^1 - v = (90 - 60)\,cm = 30cm$

The magnification now becomes $m^1 = \frac{v^1}{u^1} = \frac{90}{18} = 5$

Exercise: 6

1. A block of glass is 5.8cm thick. A point particle at its lower surface is viewed from above. The particle appears to be 3.9cm near. Calculate the refractive index of this glass. **An(1.49)**
2. A mark is made at the bottom of the beaker. Water of refractive index 1.33 is poured into the beaker to level of 5cm, to the water in the beaker is added a liquid which does not mix with water up to level of 8cm above the bottom of the beaker. When viewed normally form above, the mark appears to be $6.2cm$ below the upper level of the liquid, calculate the refractive index of the liquid added to the water. **An(1.23)**
3. A cube of glass $15cm$ thick is placed in water of refractive index 1.33 in an open container so that the upper surface of the cube is parallel to water surface of depth 10cm. a scratch at the bottom of the

cube appear to be $17.5cm$ below the water surface when viewed from vertically above. Calculate the refractive index of glass **An(1.5)**

4. A microscope is focused on a scratch on the bottom of the beaker. Turpentine is poured into the beaker to depth of 4cm and it is found to raise the microscope through a vertical distance of 1.28cm to bring the scratch back into focus. Find the refractive index of turpentine. **An(1.47)**
5. A microscope is first focused on a scratch on the inside of the bottom of an empty glass dish. water is then poured in and it is found that the microscope has to be raised by 1.2cm for refocusing. Chalk dust is sprinkled on the surface of water and this dust comes into focus when the microscope is raised an additional 3.5cm. Find the refractive index of water **An(1.34)**

Determination of refractive index of a glass block using real and apparent depth (travelling microscope)

- A cross is made on a sheet of white paper and the paper is placed under a travelling microscope
- The microscope is adjusted until the cross is focused clearly. The reading on the microscope scale is taken, $a\ cm$
- The test glass block is placed on the paper and the microscope adjusted again until the cross is clearly seen. The scale reading is recorded $b\ cm$
- Lycopodium powder is now sprinkled at the bottom of the glass block. The microscope is again adjusted until the particle are seen clearly, the scale reading $c\ cm$ is recorded
- Refractive index n is calculated from $n = \frac{c-a}{c-b}$

Determination of refractive index of a liquid using real and apparent depth (travelling microscope)

- A scratch is made at the bottom of a beaker and the beaker is placed under a travelling microscope
- The microscope is adjusted until the scratch is focused clearly. The reading on the microscope scale is taken, $a\ cm$
- The test liquid is poured in the beaker and the microscope adjusted again until the scratch is clearly seen. The scale reading is recorded $b\ cm$
- Some particles that can float on surface of the liquid are sprinkled on the liquid surface. The microscope is again adjusted until the particle are seen clearly, the scale reading $c\ cm$ is recorded
- Refractive index n is calculated from $n = \frac{c-a}{c-b}$

EXAMPLE:

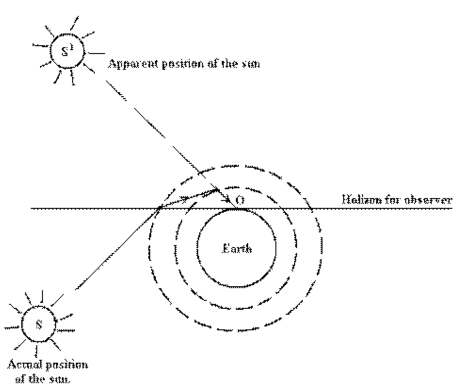
A microscope is focused on a mark at the bottom of the beaker. Water is poured in to the beaker to a depth of **8cm** and it is found necessary to raise the microscope through a vertical distance of **2cm** to bring the mark again in to focus. Find the refractive index of water.

Solution

Using the relation $n = \frac{c}{c-b} = \frac{8}{8-2} = 1.33$

Explanation of Some effects of refraction

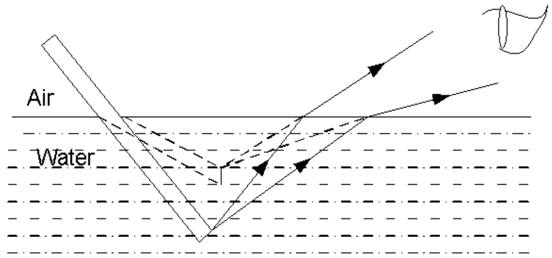
1. **Appearance of the sun when setting in the West**



- As the sun sets, layers of air near the earth get cooler and therefore more denser than layers of air higher up. Light from the sun is therefore continuously refracted towards the normal
- Rays of light which would propagate away from the earth are therefore refracted on to the earth
- When received by an observer on earth, they give an impression of presence of the sun above the horizon

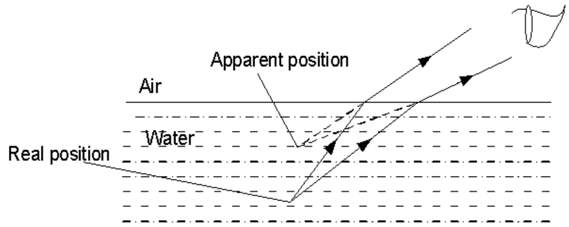
  A similar effect is seen when the sun is rising in the morning.

2. **A stick partially immersed in water**



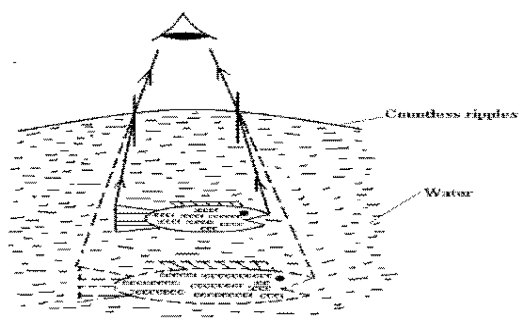
Any stick held in a slanting position in water appears to be bent.
When light rays pass from an object under water to air i.e. from a denser medium to a less dense medium, they are refracted away from the normal. As a result part of the stick under H_2O appears to be raised up. The part outside is seen in its normal position and the end result is that the stick appears

3. **A pond appears shallower than it really is**



Light rays proceeding from an object at the bottom of the pond travel from water to air. They bend away from the normal and reach the observer's eye. To an observer, the object (at the bottom of the pond) appears to be in the same straight line along which it entered his eyes, so it appears to be raised and the pond appears shallower than it really is.

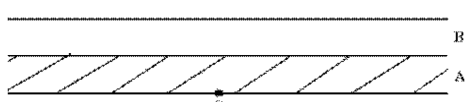
THE APARENT SIZE OF A FISH SITUATED IN WATER.



A large surface of water is not completely flat but consists of count less ripples whose convex surface on air acts as a convex lens of long focal length. In consequence the fish is with in the focal length of the lens hence it appears magnified to an observer viewing it from above.

EXERCISE: 7

1. Show that for an object viewed normally from above through a parallel sided glass block, the refractive index of the glass material is given by
$$n_g = \frac{real\ depth}{apparent\ depth}$$
2. Derive an expression for the apparent displacement of an object when viewed normally through a parallel sided glass block.
3. A vessel of depth **2d cm** is half filled with a liquid of refractive index μ_1, and the upper half is occupied by a liquid of refractive index μ_2. Show that the apparent depth of the vessel, viewed perpendicularly is $\left(\frac{1}{\mu_1} + \frac{1}{\mu_2}\right) d$
4. Two parallel sided blocks **A** and **B** of thickness **4·0cm** and **5·0cm** respectively are arranged such that **A** lies on an object **O** as shown in the figure below



Calculate the apparent displacement of **O** when observed directly from above, if the refractive indices of **A** and **B** are **1·52** and **1·66**.
5. A tank contains liquid **A** of refractive index **1·4** to a depth of **7·0cm**. Upon this floats **9·0cm** of liquid **B**. If an object at the bottom of the tank appears to be **11·0cm** below the top of liquid **B** when viewed directly above from, calculate the refractive index of liquid **B**.
6. Describe how the refractive index of a small quantity of a liquid can be determined using a concave mirror.

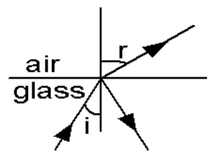
7. Describe how the refractive index of a glass block can be determined using the apparent depth method.
8. A small liquid quantity is poured into a concave mirror such that an object held above the liquid coincides with its image when it is at a height **h** from the pole of the mirror. If the radius of curvature of the mirror is **r**, show with the aid of a suitable illustration, that the refractive index of the liquid $n = \frac{r}{h}$
9. Explain how light from the sun reaches the observer in the morning before the sun appears above the horizon
10. Explain the apparent shape of the bottom of a pool of water to an observer at the bank of the pool.
11. Explain why a fish appears bigger in water than its actual size when out of water.

Total Internal Reflection and Critical angle

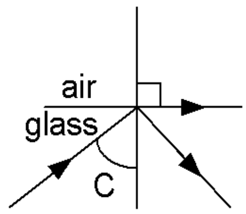
Total internal reflection if light is a reflection within the dense medium when the angle of incidence exceeds the critical angle of the medium.

Explanation of Total Internal Reflection and Critical angöle

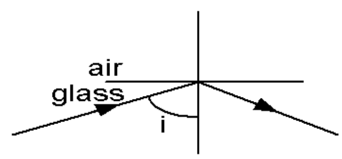
- When light travels from a more optically dense medium to a less optically dense medium, some light is reflected and some is refracted with the refracted beam being bright



- When the angle of incidence, i, is gradually increased, the angle of refraction, r also increases. At a certain value of angle of incidence the angle of refraction becomes 90° and the ray grazes the air – glass boundary. The angle of incidence for which the angle of refraction is 90° is called **critical angle(C)**



- If the angle of incidence is further increased beyond the critical angle, the light ray becomes totally reflected back to the more optically dense medium



Critical angle, C

Critical angle ,C of the medium is the angle of incidence for which the angle of refraction is 90° for a ray of light travelling from a more optically dense medium to a less optically dense medium.

Condition for total internal reflection to occur

(i) The ray of light must travel from a more dense medium to less dense eg from glass to water.
(ii) The angle of incidence must be greater than the critical angle of the medium.

Relationship between critical angle, C , and the refractive index, n

$n_g sin i = n_a \sin r$
$n sin c = 1 x \sin 90$

$$Sin\ C = \frac{1}{n}$$

Example

1. The refractive index of glass is 1.5, find the critical angle

Solution

$n_g sin\ c = n_a sin 90$
$1.5 sin\ c = 1$

C = $\sin^{-1}\left(\frac{1}{1.5}\right)$ = 41.8°

2. Find the critical angle for the ray of light moving from water of refractive index 1.33 to air

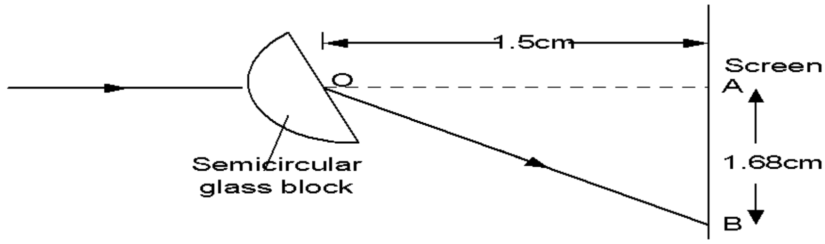
Solution

$Sin\,C = \frac{1}{n}$

$C = \sin^{-1}\left(\frac{1}{1.33}\right)$

$C = 40.75°$

3. The figure below shows monochromatic light x incident towards a vertical screen



When the semi circular glass block is placed across the path of light with its flat face parallel to screen, bright spot is formed at A. when the glass block is rotated about horizontal axis through O, the bright spot moves downwards from A towards B then just disappears at B a distance 1.68cm from A.

(i) Explain why the bright spot disappears

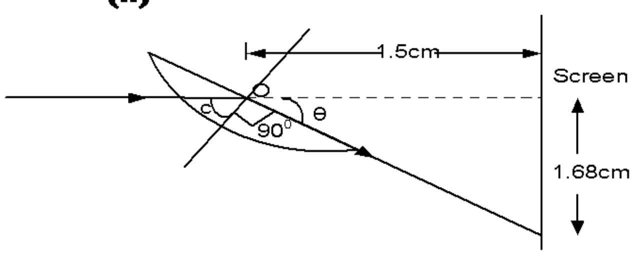
(ii) Find the refractive index of the material of glass block

(iii) Explain whether AB will be longer if the block of glass of higher refractive index was used

Solution

(i) The spot disappears because the angle of incidence has just exceeded the critical angle

(ii)



$tan\theta = \frac{1.68}{1.5}$

$\theta = tan^{-1}\left(\frac{1.68}{1.5}\right) = 48.2°$

$c + 90 + 48.2 = 180$

$c = 41.8°$

$n_g sin\, c \ = \ n_a sin 90$

$n_g sin\, 41.8 = 1$

$n_g = \frac{1}{sin 41.8} = 1.5$

(iii) The distance will be longer because a ray that moves in a more dense medium takes a longer time in the medium

APPLICATION OF TOTAL INTERNAL REFLECTION.

(i) It is responsible for the formation of a mirage.

(ii) It is responsible for the formation of a rainbow.

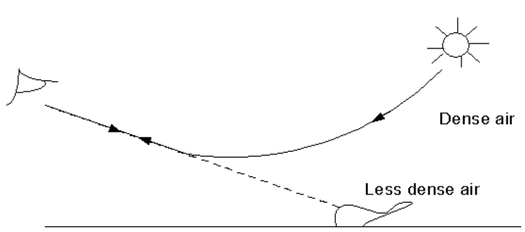
(iii) It is responsible for the transmission of light in optical fibres.

(iv). It is responsible for the transmission of sky radio waves

(v). It is responsible for the transmission of light in prism binoculars.

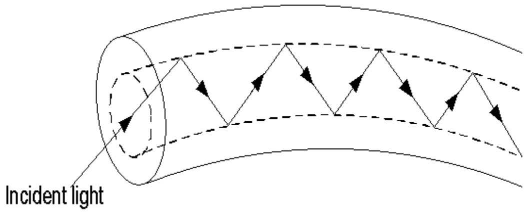
FORMATION OF A MIRAGE

Mirage: is an optical illusion of a pool of water appearing on a hot road surface



- On a hot day, The air layers near the earth's surface are hot and are less denser than the air layers above the earth's surface.
- Therefore as light from the sky pass through the various layers of air, light rays are continually refracted away from the normal till some point where light is totally internally reflected.
- An observer on earth receiving the totally internally reflected light gets an impression of a pool of water on the ground and this is the virtual image of the sky.

AN OPTICAL FIBRE



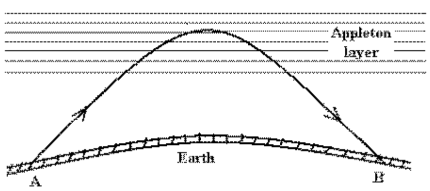
- An Optical cable is made of a transparent material coated with anther of less otptical density.
- Light entering the pipe strikes the boundary of he media at angle of incidence greater than the critical angle.
- Total internal reflection takes. This takes place repeatedly until in the pipe until the light becomes emergent form the pipe.

NOTE

(i) An optical fibre finds a practical application in an endoscope, a device used by doctors to inside the human body.

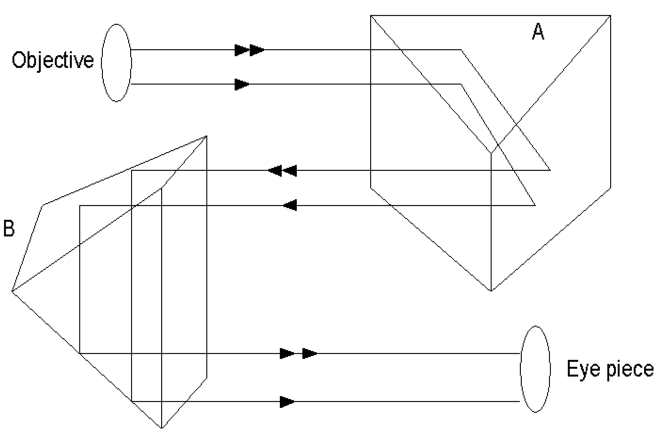
(ii) Optical fibres are used in telecommunication systems (i.e. Telephone or TV signals are carried along optical fibers by laser light).

SKY RADIO WAVES



- ❖ A radio wave sent skyward from a station transmitter **A** is continually refracted away from the normal on entering the ionosphere(Appleton layer) that exists above the earth's surface.
- ❖ Within the ionosphere(Appleton layer), the wave is totally internally reflected causing it to emerge from the ionosphere(Appleton layer) and finally returns to the earth's surface where it's presence can be detected by a radio receiver at B

Prisms binoculars

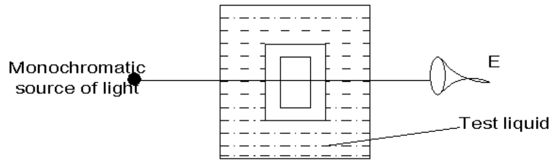


- ❖ A and B are isosclese right angled totally reflecting prisms
- ❖ A causes lateral inversion of the image formed by the objective
- ❖ B inverts the image vertically so that the final image is the same way up and same way round as object
- ❖ A and B reflect the light each through 180° making the effective length of the telescope three times the distance between the object and the eye piece

Advantages of prisms over plane mirrors

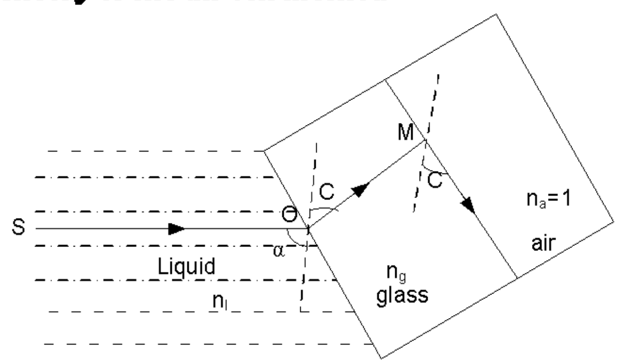
- ❖ Silvering on the plane mirror wears off with time while no silvering is required in a prism
- ❖ There is no loss of energy in prisms while thre is loss of energy at plane mirrors
- ❖ Plane mirrors form multiple images while prisms do not

Determination of refractive of a liquid using air cell method



- The liquid is poured into a parallel sided transparent vessel
- Light from monochromatic source is made incident normally on the vessel and viewed from the opposite side. The air cell is placed in the vessel so that it is illuminated normally on one side
- The position of the air cell, T_1 is noted. The air cell is now rotated in one direction keeping the light in view until light is suddenly cut off from the observer
- The angle, θ_1 of rotation of the air cell is noted
- The air cell is restored to position, T_1 . It is again rotated in opposite direction until light is suddenly cut off.
- The angle, θ_2 of rotation of the air cell is noted
- The average angle of rotation $\theta = \frac{\theta_1 + \theta_2}{2}$
- Refractive index of liquid is got from $n_l = \frac{1}{sin\theta}$

Theory of the air cell method



When the light is first cut off from the observer, it first grazes the glass air boundary as above.
From sneils law

$$n_l sin\alpha = n_g sinC = n_a sin90 = 1$$

$$n_l sin\alpha = 1$$

$$n_l = \frac{1}{sin\alpha}$$

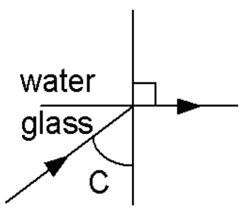
The angle between the two positions, $\theta = 2\alpha$

$$n_l = \frac{1}{sin\theta/_2}$$

Examples:

1. The critical angle for water-air interface is $48°42^{11}$ and that of glass-air interface is $38°47^1$. Calculate the critical angle for glass-water interface.

Solution



$n_g sin\ c\ = n_w\ sin\ 90º$..............................(i).

Given that for water-air interface $C_w\ = 48°42^1$.

$$\therefore n_w\ sin\ C_w = 1$$

$$n_w\ sin\ \left(48 + \frac{42}{60}\right) = 1$$

$\Rightarrow\quad n_w = 1{\cdot}33$-----------------------(ii)

Also for glass-air interface $C_g\ = 38°47^1$

$$n_g\ sin\ C_g = 1$$

$$n_g\ sin\ \left(38 + \frac{47}{60}\right) = 1$$

$\Rightarrow\quad n_g\ = 1{\cdot}67$-----------------------(iii)

Substituting equation (ii) and (iii) into equation (i) gives

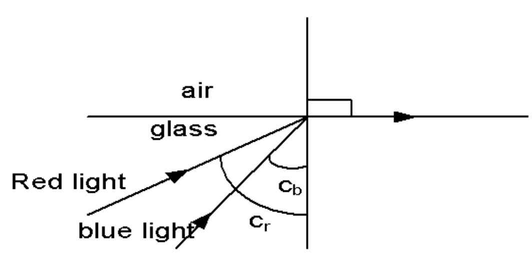
$$1{\cdot}67 sin\ c\ = 1{\cdot}33\ sin\ 90°$$

$$\Rightarrow\quad c\ =\ 52{\cdot}8°$$

2. The refractive index for red light is 1·634 of crown glass and the difference between the critical angles of red and blue light at the glass-air interface is $0°56^1$.What is the refractive index of crown glass for blue light

Solution

Analysis the critical angle between two media for red light is greater than that for any other light colour. This gives rise to the ray diagram below



Since $C_r > C_b$, then $C_r\ -\ C_b\ =\ 0°56^1$-------(i)

Applying Snell's law to red light gives

$n_r \sin C_r = 1$

$1.63 \sin C_r = 1$

$\Rightarrow \quad C_r = 37{\cdot}73^\circ$

Equation (i) now becomes

$C_r - C_b = 0°56^1$

$37{\cdot}73^\circ - C_b = \left(0 + \frac{56}{60}\right)$

$C_b = 36{\cdot}8^\circ$

Applying Snell's law to blue light gives

$n_b \sin C_b = 1$

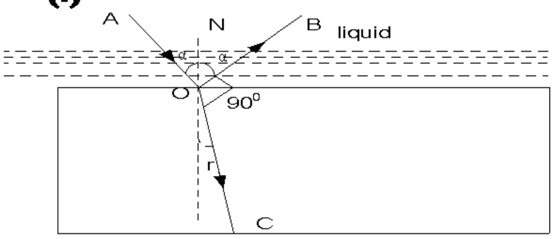
$n_b \sin 36.8 = 1$

$\Rightarrow \quad n_b = 1{\cdot}67$

4. A glass block of refractive index n_g is immersed in a liquid of refractive index n_l. A ray of light is partially reflected and partially refracted at interface such that the angle between the reflected ray and refracted ay is $90°$.
   (i) Show that $n_g = n_l \tan\alpha$ where α is the angle of incidence at the liquid glass- interface
   (ii) When the above procedure is repeated with the liquid removed, the angle of incidence increases by $8°$. find α and n_g given that $n_l = 1.33$

Solution

(i)



From 2nd law of reflection $< AON =< NOB = \alpha$

$n_l sin\alpha = n_g sinr$

$$n_g = \frac{n_l sin\alpha}{sinr}$$

But $r + 90° + \alpha = 180°$

$r = 90° - \alpha$

$$n_g = \frac{n_l sin\alpha}{sin(90° - \alpha)} = \frac{n_l sin\alpha}{cos(\alpha)} = n_l\, tan\alpha$$

(ii) From $n_g = n_l \tan\alpha$ ---------------------(i)

When liquid is removed $n_l = n_a = 1$

$\Rightarrow \quad n_g = 1xtan(\alpha + 8)$

$$\therefore\ n_g = \frac{\tan\alpha + \tan 8}{1 - \tan 8 \tan\alpha}$$

$n_g\,(1 - \tan 8 \tan\alpha) = \tan\alpha + \tan 8$

$n_g - n_g tan\,\alpha \tan 8° = \tan\alpha + \tan 8°$------(ii)

from equation (i) $\tan\alpha = \frac{n_g}{n_l}$

Substituting for tan α in equation (ii) gives

$$n_g - \frac{n_g{}^2 \tan 8}{n_l} = \frac{n_g}{n_l} + \tan 8°$$

but $n_l = 1{\cdot}33$.

$\therefore n_g{}^2 - 2.340\, n_g + 1{\cdot}326 = 0$ ------------(iii)

Equation (iii) is quadratic in n_g and solving it gives

$n_g = 1{\cdot}39$ **or** n_g not physically possible.

Using equation (i)

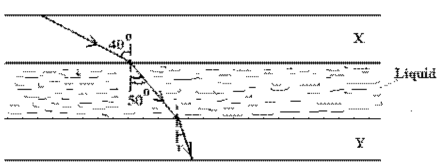
$$\tan\alpha = \frac{n_g}{n_l} = \frac{1.39}{1.33}$$

$\alpha = \tan^{-1}(1.045) = 46{\cdot}3°$

The required angle of incidence $= \alpha + 8°$

$= \mathbf{54{\cdot}3°}$

5. The figure below shows a liquid layer confined between two transparent plates **X** and **Y** of refractive index 1.54 and 1.44 respectively.



A ray of monochromatic light making an angle of **40**° with the normal to the interface between media **X** and the liquid is refracted through an angle of **50**° by the liquid. Find the
   (i) refractive index of the liquid.
   (ii) angle of refraction ,**r** in the medium **Y.**
   (iii) minimum angle of incidence in the medium **X** for which the light will not emerge from medium **Y.**

Solution

(i) Applying Snell's law at the plate **X** – liquid interface gives

$n_x sini = n_l sinr$

$1.54 sin40 = n_l \sin 50$

$\therefore\ n_l = \mathbf{1.29}$

(ii) Applying Snell's law at the liquid – plate **Y** interface gives

$$n_l sin i = n_y \sin r$$

$$1.29 sin 50 = n_l \sin r$$

$$\Rightarrow \quad \angle r \ = \ 43.3^\circ$$

(iii) For light not to emerge from plate Y, it grazes the liquid – plate Y interface.

$\Rightarrow \quad \angle r = 90°$

Applying Snell's law at the liquid – plate Y interface gives

$$n_l sin i_l = n_y \sin r$$

$$1.29 sin i_l = 1.44 \sin 90$$

$$sin i_l = \frac{1.44}{1.29} \text{ -----------------(i)}$$

More over, applying Snell's law at the plate X – liquid interface gives

$$n_x sin i_x = n_l \sin i_l$$

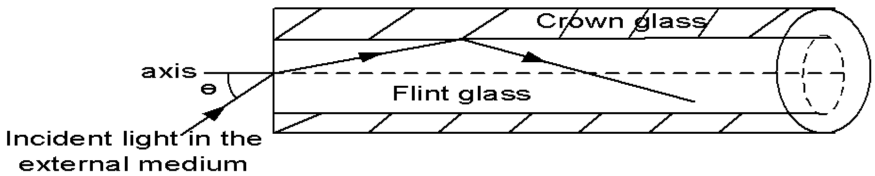
$$1.54 sin i_x = 1.29 \sin i_l \text{--------------(ii)}$$

Substituting equation (i) in (ii) gives

$$1.54 sin i_x = 1.29 x \frac{1.44}{1.29}$$

$$\Rightarrow \quad \angle i_x = 40.5^\circ$$

6. The diagram below shows a cross-section through the diameter of the light pipe with an incident ray of light in its plane.



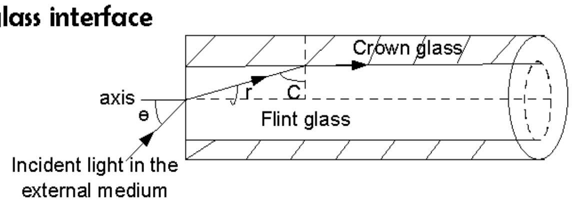
The refractive indices for flint glass, crown glass and the external medium are n_1 , n_2 and n_3 respectively. Show that a ray that enters the pipe is totally reflected at the flint-crown glass interface provided

$$\sin\theta = \frac{\sqrt{n_1^2 - n_2^2}}{n_3}$$

Where θ is the maximum angle of incidence in the external medium.

Solution

Analysis for light to be totally reflected, it must be incident at a critical angle on the flint-crown glass interface



Applying Snell's law at the external medium-flint glass interface gives

$$n_3 \sin\theta \ = \ n_1 sin\, r$$

but $r + c \ = \ 90^\circ$

$$\Rightarrow \quad n_3 \ sin\,\theta \ = \ n_1 sin\,(90^\circ - c\,)$$

$$\therefore \quad n_3 \ sin\,\theta \ = \ n_1 \ cos\, c$$

$$\Rightarrow \quad cos\ c = \frac{n_3 \ sin\,\theta}{n_1} \text{ -----------------(i)}$$

Applying Snell's law at the flint-crown glass interface gives

$$n_1 \ sin\, C \ = \ n_2 sin\, 90$$

$$\Rightarrow \quad sin\, C \ = \frac{n_2}{n_1} \text{ ---------------------------------(ii)}$$

Using the trigonometrical relation

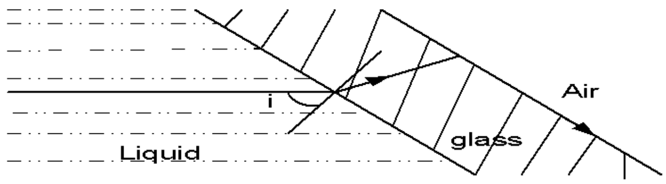
$sin^2 c \ + \ cos^2 c \ = 1$, then

$$\left(\frac{n_2}{n_1}\right)^2 + \left(\frac{n_3 \ sin\theta}{n_1}\right)^2 = 1$$

$$\sin\theta = \frac{\sqrt{n_1^2 - n_2^2}}{n_3}$$

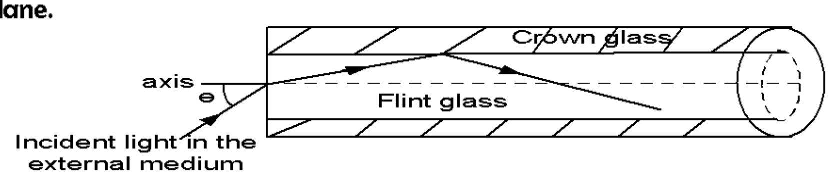
Exercise:8

1. Explain the term total internal reflection and give three instances where it is applied.
2. With the aid of suitable ray diagrams, explain the terms critical angle and total internal reflection.
3. Show that the relation between the refractive index **n** of a medium and critical angle **c** for a ray of light traveling from the medium to air is given by $n = \frac{1}{Sin\ C}$
4. Show that the critical angle, **c** at a boundary between two media when light travels from medium 1 to medium 2 is given by $Sin\ C = \frac{n_2}{n_1}$ where n_1 and n_2 are the refractive indices of the media respectively.
5. Explain how a mirage is formed.
6. Explain briefly how sky radio waves travel from a transmitting station to a receiver.
7. Describe how you would determine the refractive index of the liquid using an air cell.
8.



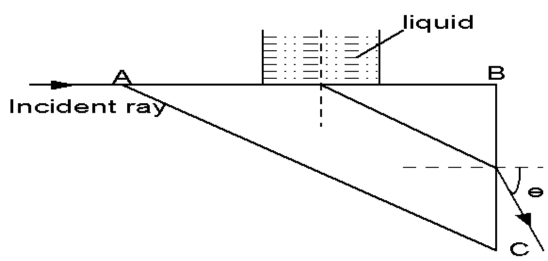
In the figure above, a parallel sided glass slide is in contact with a liquid on one side and air on the other side. A ray of light incident on glass slide from the liquid emerges in air along the glass-air interface. Derive an expression for the absolute refractive index **,n,** of the liquid in terms of the angle of incidence **i** in the liquid-medium.

9. The diagram below shows a cross-section through the diameter of the light pipe with an incident ray of light in its plane.



The refractive indices for flint glass and crown glass are n_1 and n_2 respectively. Show that a ray which enters the pipe is totally reflected at the flint-crown glass interface provided $\sin\theta = \sqrt{n_1^2 - n_2^2}$ where θ is maximum angle of incidence at the air-flint glass interface

10. A liquid of refractive index n_l is tapped in contact with the base of a right-angled prism of refractive index n_g by means of a transparent cylindrical pipe as shown.

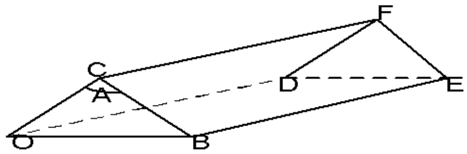


Show that a ray of light which is at a grazing incidence on the liquid-glass interface emerges in to air through face **BC** at an angle θ below the horizontal provided $n_l = \sqrt{n_g{}^2 - sin^2\theta}$ **.** Hence find n_l**,** if $n_g = 1.52$ and $\theta = 47.4^o$

REFRACTION IN A GLASS PRISM

A prism is a geometrical object with at least two plane surfaces. A prism is made up of glass.

Terms used in a prism



- Triangular face; OBC=DEF
- Refracting surface; CBEF= CODF

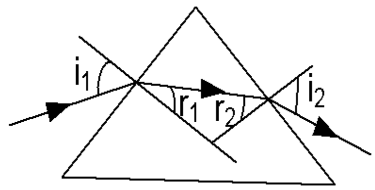
- Angle of a prism or refracting angle;

This is the angle between any two inclined surfaces of a prism and its denoted by A

Eg $< OCB =< DFE = A$

- Base; OBED

Path followed by a ray of light in a glass prism



Explain why a prism deviates light towards the base

This is because a ray that moves form a less optically dense medium bends towards the normal and a ray that moves from a more optically dense medium bends away from the normal

Deviation by a prism

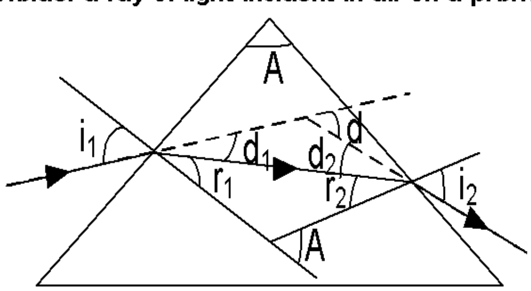
It is the change in direction of a ray of light produced by a prism.

When light passes through a glass prism, the direction of the emergent ray is altered from the initial direction. The angle through which the beam direction is altered is called **deviation, d**

Definition

The angle of deviation caused by the prism is the angle between the incident ray and the emergent ray.

Consider a ray of light incident in air on a prism of refracting angle **A** and finally emerges into air as shown



$$\boldsymbol{d = d_1 + d_2}$$

Since $d_1 = i_1 - r_1$ and $d_2 = i_2 - r_2$

$$\boldsymbol{d = (i_1 - r_1) + (i_2 - r_2)}$$

$$\boxed{\boldsymbol{d = (i_1 + i_2) - (r_1 + r_2)}}$$

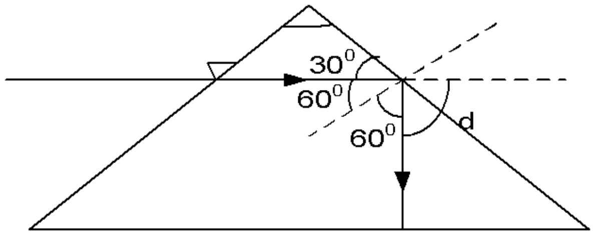
Since $A = r_1 + r_2$

$$\boxed{\boldsymbol{d = (i_1 + i_2) - A}}$$

Examples

1. A beam of monochromatic light is incident normally on glass prism of refractive angle $60°$. If the refractive index of glass is 1.62, calculate the deviation caused by the prism

Solution



$$n_g sinc = 1$$

$$c = sin^{-1}\left(\frac{1}{1.62}\right) = 38°$$

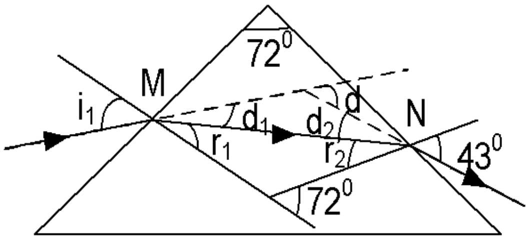
The angle of incidence at the second surface is $60°$ which is greater than the critical angle 38, hence total internal reflection will occur

Total deviation $d = 30° + 30° = 60°$

2. A ray of light is incident on a prism of refracting angle **72°** and refractive index of **1.3.** The ray emerges from the prism at **43°.** Find

   (i) the angle of incidence.
   (ii) the deviation of the ray.

Solution



(i) Applying Sneil's law at N :

$$n\ Sin\ i = constant$$
$$n_g \sin r_2 = n_a \sin 43$$
$$1.3 \sin r_2 = 1x \sin 43$$
$$r_2 = sin^{-1}\left(\frac{sin43}{1.3}\right)$$
$$r_2 = 31.64°$$
$$A = r_1 + r_2$$
$$r_1 = 72° - 31.64°$$
$$r_1 = 40.36°$$

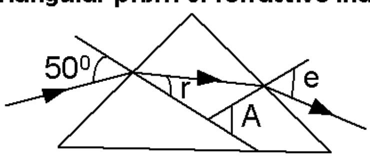
Applying Sneil's law at M

$$n\ Sin\ i = constant$$
$$n_g \sin r_1 = n_a \sin i_1$$
$$1.3x \sin 40.36° = 1x \sin i_1$$
$$i_1 = sin^{-1}\left(\frac{1.3xsin40.36}{1}\right)$$
$$i_1 = 57.34°$$

(ii) $\boldsymbol{d = (i_1 + i_2) - A}$

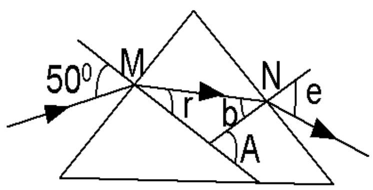
$$d = (57.34° + 43) - (72°)$$
$$d = 28.34°$$

3. The diagram below shows a ray of monochromatic light incident at an angle of 50° on an equilateral triangular prism of refractive index 1.52



(i) Calculate the angles marked **r** and **e**
(ii) Find the deviation produced
(iii) Explained what could be observed if the ray above were of white light

Solution



i) Applying Sneil's law at M

$$n\ Sin\ i = constant$$
$$n_a \sin 50 = n_g \sin r$$
$$1x \sin 50 = 1.52x \sin r$$
$$r = sin^{-1}\left(\frac{sin50}{1.52}\right)$$
$$r = sin^{-1}\left(\frac{0.766}{1.52}\right)$$
$$r = sin^{-1}(0.504)$$
$$r = 30.26°$$

Since it equilateral, $A = 60°$

$A = r + b$ hence $b = A - r$

$$b = 60° - 30.26°$$
$$b = 29.74°$$

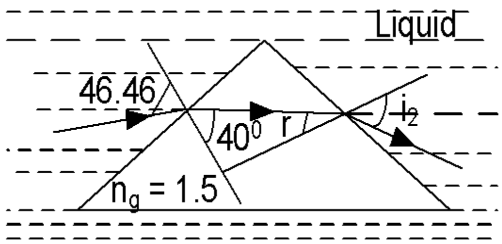
Applying Sneil's law at N

$$n\ Sin\ i = constant$$
$$n_g \sin 50 = n_a \sin r$$
$$1.52x \sin 29.74 = 1x \sin e$$
$$e = sin^{-1}\left(\frac{1.52xsin29.74}{1}\right)$$
$$e = sin^{-1}\left(\frac{1.52\ x\ 0.4961}{1}\right)$$
$$e = sin^{-1}(0.754)$$
$$e = 48.94°$$

ii) $\boldsymbol{d = (i_1 + i_2) - A}$

$$d = (50° + 48.94°) - (60°)$$
$$d = 38.94°$$

4. The diagram below shows a ray of light incident at an angle of $46.46°$ on one side of an equilateral triangular prism immersed in liquid of refractive index n_l,



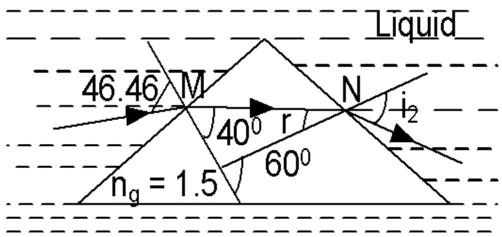
Given that the refractive index of glass is 1.5 and the angle of refraction at the first face is $40°$, calculate

(i) The value of refractive index of the liquid

(ii) The value of i_2 and

(iii) The angle of deviation

Solution



i) Applying Sneil's law at M

$$n\,Sin\,i = constant$$
$$n_l \sin 46.46 = n_g \sin 40$$
$$n_l\,x \sin 46.46 = 1.5x \sin 40$$
$$n_l = \left(\frac{1.5x0.6428}{0.7249}\right)$$
$$n_l = 1.33$$

ii) Since it equilateral, $A = 60°$

$A = r + 40°$ **hence** $r = A - 40°$

$$r = 60° - 40°$$
$$r = 20°$$

Applying Sneil's law at N

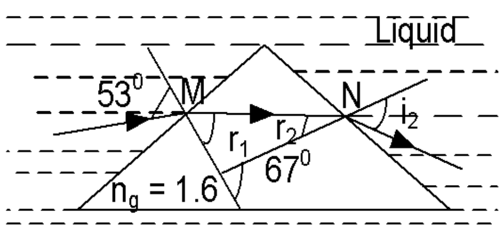
$$n\,Sin\,i = constant$$
$$n_g \sin 20 = n_l sini_2$$
$$1.5x \sin 20 = 1.33xsini_2$$
$$i_2 = sin^{-1}\left(\frac{1.5xsin20}{1.33}\right)$$
$$i_2 = sin^{-1}\left(\frac{1.5\,x\,0.342}{1.33}\right)$$
$$i_2 = sin^{-1}(0.3857)$$
$$i_2 = 22.69°$$

iii) $\boldsymbol{d = (i_1 + i_2) - A}$

$$d = (46.46° + 22.69°) - (60°)$$
$$d = 9.15°$$

4. **A prism of refracting angle 67° and refractive index of 1·6 is immersed in a liquid of refractive index 1·2. If a ray of light traveling through the liquid makes an angle of incidence of 53° at the left face of the prism,** Determine the total deviation of the ray

Solution



iii) Applying Sneil's law at M

$$n\,Sin\,i = constant$$
$$n_l \sin 53 = n_g \sin r_1$$
$$1.2x \sin 53 = 1.6x \sin r_1$$
$$r_1 = sin^{-1}\left(\frac{1.2xsin53}{1.6}\right)$$
$$r_1 = 36.8°$$

$$A = r_1 + r_2$$
$$r_2 = 67° - 36.8°$$
$$r_2 = 30.2°$$

Applying Sneil's law at N

$$n\,Sin\,i = constant$$
$$n_g \sin 36.8 = n_l sini_2$$
$$1.6x \sin 36.8 = 1.2xsini_2$$
$$i_2 = sin^{-1}\left(\frac{1.6xsin36.8}{1.2}\right)$$
$$i_2 = 42.12°$$

iv) $\boldsymbol{d = (i_1 + i_2) - A}$

$$d = (53° + 42.12°) - (67°)$$
$$d = 28.12°$$

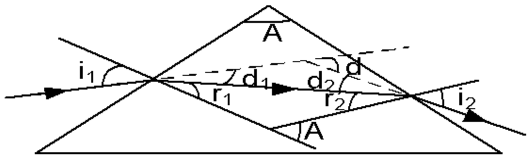
Minimum Deviation by a prism

Minimum deviation occurs when;

- The ray of light passes symmetrically
- The angle of incidence must be equal to the angle of emergence i.e $i_1 = i_2 = i$ and $r_1 = r_2 = r$

Relation of angle of prism A, minimum deviation and refractive index

Consider a ray on one face of the prism at an angle **i_1** and leaves it at an angle **i_2** to the normal as shown



$$d = d_1 + d_2$$

Since $d_1 = i_1 - r_1$ and $d_2 = i_2 - r_2$

$$d = (i_1 - r_1) + (i_2 - r_2)$$

$$\boxed{d = (i_1 + i_2) - (r_1 + r_2)}$$

Since $A = r_1 + r_2$

$$d = (i_1 + i_2) - A$$

But for minimum deviation $i_1 = i_2 = i$ and

$$r_1 = r_2 = r$$

$$d_{min} = 2i - A \quad \therefore \quad i = \frac{d_{min}+A}{2}$$

Also $A = r_1 + r_2 = 2r$

$$r = \frac{A}{2}$$

$$n_a \sin i = n_g sinr$$

$$n_g = n_a \frac{\sin(i)}{sin(r)}$$

$$n_g = n_a \frac{\sin\left(\frac{d_{min} + A}{2}\right)}{sin\left(\frac{A}{2}\right)}$$

But $n_a = 1$

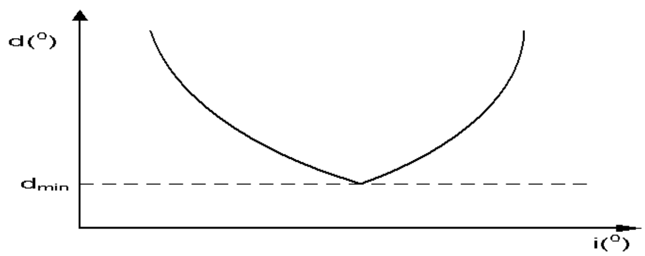
$$\boxed{n_g = \frac{\sin\left(\frac{d_{min} + A}{2}\right)}{sin\left(\frac{A}{2}\right)}}$$

Note: If the prism is surrounded by a medium of refractive index n_l

$$\boxed{n_g = n_l \frac{\sin\left(\frac{d_{min} + A}{2}\right)}{sin\left(\frac{A}{2}\right)}}$$

A graph of deviation against angle of incidence

Experiments show that as the angle of incidence **i** is increased from zero, the deviation **D** reduces continuously up to a minimum value of deviation **D_{min}** and then increases to a maximum value as the angle of incidence is increased as shown below:



Examples

1. Calculate the angle of incidence at minimum deviation for light passing through a Prism of refracting angle **70°** and refractive index of **1·65**.

Solution

$$n_g = n_a \frac{\sin\left(\frac{d_{min} + A}{2}\right)}{sin\left(\frac{A}{2}\right)}$$

$$1.65 = 1x \frac{\sin\left(\frac{d_{min} + 70}{2}\right)}{sin\left(\frac{70}{2}\right)}$$

$$d_{min} = 72.33°$$

$$i = \frac{d_{min} + A}{2}$$

$$i = \frac{72.33 + 70}{2} = 71.17°$$

2. An equilateral glass prism of refractive index **1·5** is completely immersed in a liquid of refractive index **1·3**. if a ray of light passes symmetrically through the prism, calculate the:
   (i) angle of deviation of the ray.
   (ii) angle of incidence

Solution

(a) For an equilateral prism, its refracting angle A =60°

(b) If the ray passes through the prism symmetrically, then the angle of deviation is minimum

$$n_g = n_l \frac{\sin\left(\frac{d_{min} + A}{2}\right)}{sin\left(\frac{A}{2}\right)}$$

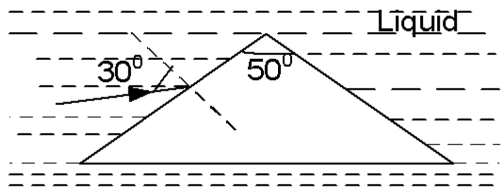
$$1.5 = 1.3x\frac{\sin\left(\frac{d_{min}+60}{2}\right)}{sin\left(\frac{60}{2}\right)}$$

$$d_{min} = 10.47°$$

$$i = \frac{d_{min}+A}{2}$$

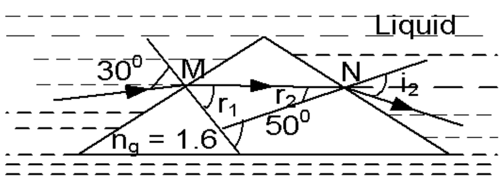
$$i = \frac{10.47+60}{2} = 35.24°$$

3. **A ray of light propagating in a ;liquid is incident on prims of refractive angle 50° and refractive index 1.6 at an angle of 30° as shown below**



If the ray passes symmetrically in the prism, find refractive index of liquid

Solution



For symmetrical ray $A = r_1 + r_2 = 2r$

$$r = \frac{A}{2} = \frac{50}{2} = 25°$$

$$i_1 = i_2 = i$$

$$n_l \sin i = n_g \sin r$$

$$n_l \sin 30 = 1.6 \sin 25$$

$$n_l = 1.35$$

Alternatively

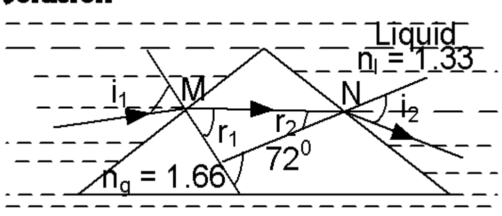
$$n_g = n_l\frac{\sin\left(\frac{d_{min}+A}{2}\right)}{sin\left(\frac{A}{2}\right)}$$

$$1.6 = n_l x\frac{\sin(30)}{sin\left(\frac{50}{2}\right)}$$

$$n_l = 1.35$$

3. **A glass of refractive angle 72° and refractive index 1.66 and it is immersed in the liquid of refractive index 1.33. Calculate;**
   (i) **Angle of incidence of ray of light, if its passes through symmetrical**
   (ii) **The minimum deviation**

Solution



For symmetrical ray $A = r_1 + r_2 = 2r$

$$r = \frac{A}{2} = \frac{72}{2} = 36°$$

$$i_1 = i_2 = i$$

$$n_l \sin i = n_g \sin r$$

$$1.33 \sin i = 1.66 \sin 36$$

$$i = 47.2°$$

$$n_g = n_l\frac{\sin\left(\frac{d_{min}+A}{2}\right)}{sin\left(\frac{A}{2}\right)}$$

$$1.33\sin\left(\frac{d_{min}+72}{2}\right) = 1.66 sin\left(\frac{72}{2}\right)$$

$$d_{min} = 22.38°$$

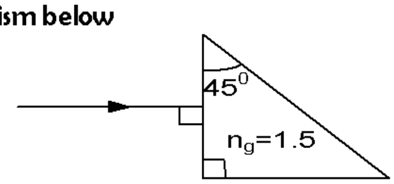
Or $d_{min} = (2i) - A$

$$d_{min} = 2x47.2 - 72$$

$$d_{min} = 22.4$$

Exercise;9

1. **Calculate the total deviation in the prism below**

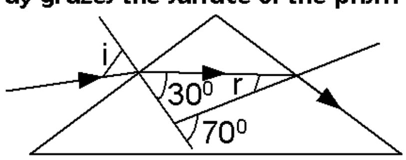


An(90°)

2. **Light propagation in air is incident at 12° on a glass prism of refractive index 1.54 and refracting angle 60 ° as show. The emergent beam glass faces of the prism in contact with the liquid. Find**
   (i) **refractive index of liquid**

(ii) deviation produced by the prism. **An(1.2, 42°)**

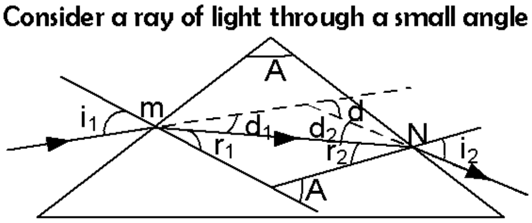
3. Light of two wave length is incident at a small angle on a thin prism of refracting angle 5° and refractive indices 1.54 and 1.48 for the two wave lengths. Find the angular separation of the two wave's length after refraction by the prism. **An(0.3°)**
4. A ray of light just undergoes total internal reflection at the second plane of the prism of refracting angle 60° and refractive index 1.5. what is its angle of incidence on the face
5. A ray of monochromatic light is incident at an angle of 30° on a prism of which the refractive index 1.52.What is the maximum refracting angle of the prism if light is just to emerge from the opposite face. **An(60.34°)**
6. Calculate the critical angle for a glass air surface if a ray of light which is incident in air is deviated through 15.5° when its angle of incidence is 40°.
7. Calculate the angular separation of the red and violet rays which image from a 60° glass prism when a ray of white light is incident on the prism at an angle of 45°. Glass has a refractive index of 1.64 for red light and 1.66 for violet light.
8. Monochromatic light is incident at an angle of 45° on a glass prism of refracting angle 70° in air. The emergent light grazes the other refracting surface of the prism. Find the refractive index of the glass. **An(1.5)**
9. Monochromatic light propagating in aliquid is incident at an angle of 40° on a glass prism of refracting angle 60° and refractive index 1.50. if the ray passes symmetrically through the prism, find the refractive index of the liquid. **An(1.2)**
10. A glass of refractive angle 60° and refractive index 1.5 and it is immersed in the liquid of refractive index 1.3. Calculate;
    (i) Angle of incidence of ray of light, if its passes through symmetrical
    (ii) The minimum deviation **An(35.2°, 10.4°)**
11. A monochromatic beam of light is incident at an angle i, on a glass prism of refracting angle $70°$, the emergent ray grazes the surface of the prism as shown below



Find ;
    (i) The refractive index of the prism
    (ii) Angle i
12. A ray of red light is incident on a prism of refractive index 1.48 and refracting angle 60°. The ray emerges from a prism at an angle of 43°. Find;
    (i) The angle of incidence
    (ii) The angle of deviation

Deviation produced by small angle prism (A$\leq 10°$)

Consider a ray of light through a small angle prism



At m: $\sin i_1 = n \sin r_1$

For small angles i_1 and r_1 measured in radians, $\sin i_1 \approx i_1$ and $\sin r_1 \approx r_1$

$$i_1 = nr_1 \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

But $r_2 = A - r_1$, since A and r_1 are both small then r_2 is also small

At N: $\sin i_2 = n \sin r_2$

Since r_2 is small angles and $\sin i_2 = n \sin r_2$ then i_2 is also small, $\sin i_2 \approx i_2$ and $\sin r_2 \approx r_2$

$$i_2 = nr_2 \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Deviation produced by prism ,

$\boldsymbol{d} = (\boldsymbol{i_1} - \boldsymbol{r_1}) + (\boldsymbol{i_2} - \boldsymbol{r_2})$

$$\boldsymbol{d}\ \boldsymbol{d} = nr_1 - r_1 + nr_2 - r_2$$

$$\boldsymbol{d} = (\boldsymbol{n} - 1)\ (\boldsymbol{r_1} + \boldsymbol{r_2})$$

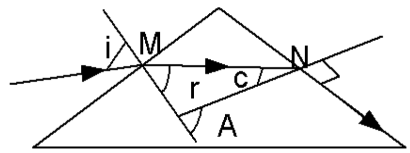
$$\boxed{\boldsymbol{d} = (\boldsymbol{n} - 1)\boldsymbol{A}}$$

GRAZING PROPERTY OF LIGHT RAYS AS APPLIED TO PRISMS.

If a ray of light is either such that the incident angle or the emergent angle is equal to 90° to the normal of the prism, then the ray is said to graze the refracting surface of the prism.

Consider a ray of light incident at an angle i on a glass prism of refracting angle A situated in air with the emergent light grazing the other refracting surface of the prism as shown.

Maximum deviation diagram



From the diagram, $r + c = A$

$\therefore\ r = A - C$---------------------------------(a)

At M Snell's law becomes

$n_a \sin i = n_g sin r$ -------------(b)

Substituting equation (a) in (b) gives

$\sin i = n_g sin(A - C)$

$\Rightarrow sin\ i = n_g\ (sin\ A\ cos\ C - sin\ C\ cos\ A)$ ------(c)

At N, Snell's law becomes

$n_g\ sin\ c = n_a\ sin\ 90°$.

$\therefore sin\ c = \frac{1}{n_g}$

But $\cos C = \sqrt{1 - sin^2 c} = \sqrt{1 - \left(\frac{1}{n_g}\right)^2} = \frac{\sqrt{(n_g{}^2 - 1)}}{n_g}$

Substituting $sin\ c$ and $cos\ c$ in equation c gives

$$sin\ i = n_g\ \left(sin\ A \left[\frac{\sqrt{(n_g{}^2 - 1)}}{n_g}\right] - \frac{1}{n_g} cos\ A\right)$$

On simplifying we have $\sqrt{(n_g{}^2 - 1)} = \frac{\sin i + cos A}{\sin A}$

Squaring both sides and simplifying for n_g gives

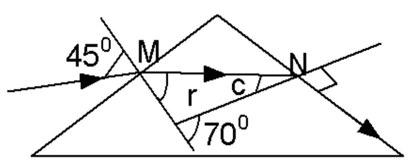
$$n_g = \sqrt{1 + \left(\frac{\sin i + \cos A}{\sin A}\right)^2}$$

Knowing the angles i and A, the refractive index n_g of a material of a prism can be determined.

EXAMPLES

13. Monochromatic light is incident at an angle of **45°** on a glass prism of refracting angle **70°** in air. The emergent light grazes the other refracting surface of the prism. Find the refractive index of the glass material.

Solution



At M, Snell's law becomes

$n_a \sin 45 = n_g sin r$ --------------------(a)

From the diagram, $r + c = 70°$

$\Rightarrow\ r = 70° - c$ ---------------(b)

Substituting equation (b) in (a) gives

$Sin\ 45° = n_g\ sin\ (70° - c)$ ----------(c)

At N, Snell's law becomes

$n_g sin\ c = n_a\ sin\ 90°$

$n_g = \frac{1}{\sin C}$---------------------------------(d)

Substituting equation (d) in (c) gives

$Sin\ 45° = \frac{sin\ (70° - c)}{\sin C}$

$sin\ 45° sin\ c = sin\ 70° cos\ c - sin\ c\ cos\ 70°$

$(sin\ 45° + cos\ 70°)\ sin\ c = sin\ 70° cos\ c$

Dividing $cos\ c$ throughout gives

$$\tan C = \frac{\sin 70}{(sin\ 45° + cos\ 70°)}$$

$$c = 41.9°$$

$$n_g = \frac{1}{\sin C}$$

$$n_g = \frac{1}{\sin 41.9}$$

$$n_g = 1.497$$

Alternatively

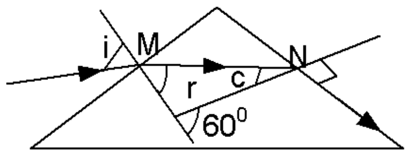
$$n_g = \sqrt{1 + \left(\frac{\sin i + \cos A}{\sin A}\right)^2}$$

$$n_g = \sqrt{1 + \left(\frac{\sin 45 + \cos 70}{\sin 70}\right)^2}$$

$$n_g = 1.497$$

14. A ray of light is incident on one refracting face of a prism of refractive index 1·5 and refracting angle 60°. Calculate the minimum angle of incidence for the ray to emerge through the second refracting face.

Solution



for minimum angle of incidence, the emergent ray grazes the second refracting face.

At **N**, Snell's law becomes

$$1{\cdot}5 \sin c = n_a \sin 90°$$
$$\Rightarrow c = 41{\cdot}8°$$

But $r + c = 60°$

$$\Rightarrow r = 60° - c$$
$$= 60° - 41{\cdot}8°$$
$$\therefore r = 18{\cdot}2°$$

At **M**, Snell's law becomes

$$1{\cdot}5 \sin 18{\cdot}2° = n_a \sin i$$
$$\therefore i = 27{\cdot}9°$$

Alternatively

$$n_g = \sqrt{1 + \left(\frac{\sin i + \cos A}{\sin A}\right)^2}$$

$$1.5 = \sqrt{1 + \left(\frac{\sin i + \cos 60}{\sin 60}\right)^2}$$

$i = 27{\cdot}9°$

EXERCISE:10

1. (i) Obtain an expression relating the deviation of a ray of light by the prism to the refracting angle and the angles of incidence and emergence.

(ii) The deviation of a ray of light incident on the first face of a **60°** glass prism at an angle of **45°** is **40°.** Calculate the angle of emergence of a ray on the second face of the prism. [**Ans i_2** = 65°]

(iii) A prism of refractive index **1·64** is immersed in a liquid of refractive index **1·4**. A ray of light is incident on one face of the prism at an angle of **40°.** If the ray emerges at an angle of **29°,** determine the angle of the prism. [**Answer: 57·7**°]

2. (i) For a ray of light passing through the prism, what is the condition for minimum deviation to occur?

(ii) Derive an expression for the refractive index of a prism in terms of the refracting angle, **A**, and the angle of minimum deviation **D**.

(iii) A glass prism of refractive index n and refracting angle ,**A**, is completely immersed in a liquid of refractive index n_l. If a ray of light that passes symmetrically through the prism is deviated through an angle φ**,** Show that

$$\frac{n_l}{n} = \frac{\sin\left(\frac{A}{2}\right)}{\sin\left(\frac{\varphi + A}{2}\right)}$$

3. (a) A glass prism with refracting angle **60°** is made of glass whose refractive indices for red and violet light are respectively 1·514 and 1·530**.** A ray of white light is set incident on the prism to give a minimum deviation for red light.

Determine the:

(i) angle of incidence of the light on the prism.

(ii) angle of emergence of the violet light.

(iii) angular width of the spectrum.

(b) A certain prism is found to produce a minimum deviation of **51°.** While it produces a deviation of **62·8**° for a ray of light incident on its first face at an angle of **40·1**° and emerges through its second face at an angle of **82·7°.**

Determine the:

(i) refracting angle of the prism.

(ii) angle of incidence at minimum deviation.

(iii) refractive index of the material of the prism.

[**Ans (i) 60°** **(ii) 55·5°** **(iii) 1·648**]

4. **(i)** A ray of monochromatic light is incident at a small angle of incidence on a small angle prism in air. Obtain the expression $D = (n - 1)A$ for the deviation of light by the prism.

   (ii) A glass prism of small angle ,**A**, and refractive index n_g and is completely immersed in a liquid of refractive index n_l. Show that a ray of light passing through the prism at a small angle of incidence suffers a deviation given by $D = \left(\frac{n_g}{n_l} - 1\right)A$

5. Explain why white light is dispersed by a transparent medium.
6. Light of two wave length is incident at a small angle on a thin prism of refracting angle 5° and refractive index of 1·52 and 1·48 for the two wave lengths. find the angular separation of the two wave lengths after refraction by the prism. [**Ans φ = 0·2°**]
7. A point source of white light is placed at the bottom of a water tank in a dark room. The light from the source is observed obliquely at the water surface. Explain what is observed.
8. Monochromatic light is incident at an angle φ on a glass prism of refracting angle ,A, situated in air. If the emergent light grazes the other refracting surface of the prism, Show that the refractive index, n_g, of the prism material is given by

$$n_g = \sqrt{1 + \left(\frac{\sin i + \cos A}{\sin A}\right)^2}$$

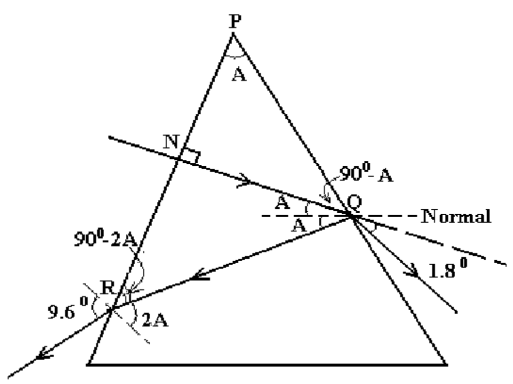
9. A ray of light is incident at angle of 30° on a prism of refractive index 1·5 .calculate the limiting angle of the prism such that the ray does not emerge when it meets the second face. [**Ans A = 61·3°**]

EXAMPLES:

A ray of light that falls normally upon the first face of a glass prism of a small refracting angle under goes a partial refraction and reflection at the second face of the prism. The refracted ray is deviated through an angle $1.8°$ and the reflected ray makes an angle of $9.6°$ with the incident ray after emerging from the prism through its first face. Calculate the refracting angle of the prism and its refractive index of the glass material.

Solution

Let **A** be the required refracting angle of the prism as shown



Consider the deviation suffered by the incident light

$D = (n - 1)A$

$\Rightarrow 1.8° = (n - 1)A$ --------------------(i)

From ΔPQN, $\angle PQN = 90° - A$

$\Rightarrow$ At Q, the angle of incidence $= A$

From ΔNQR, $\angle QRN = 90° - 2A$

$\Rightarrow$ At R, the angle of incidence $= 2A$

∴At R, Snell's becomes $n_a \sin 9{\cdot}6° = n \sin 2A$

For small angles, $\sin 9.6° \approx 9.6°$ **and** $\sin 2A \approx 2A$

$\Rightarrow 9.6° = 2nA$ ----------------------------(ii)

Equation (i) ÷ Equation (ii) gives

$$\frac{1.8°}{9.6°} = \frac{(n-1)A}{2nA}$$

$\Rightarrow 3.6°n = 9.6°(n - 1)$

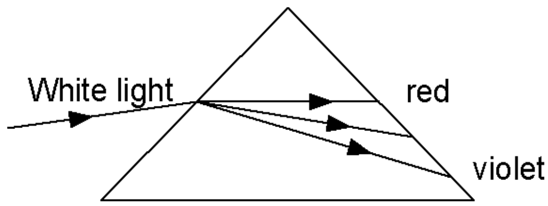
Thus $n = 1.6$

Equation (i) now becomes $1.8° = (1.6 - 1)A$

$\therefore A = 3°$

DISPERSION OF WHITE LIGHT BY A TRANSPARENT MEDIUM

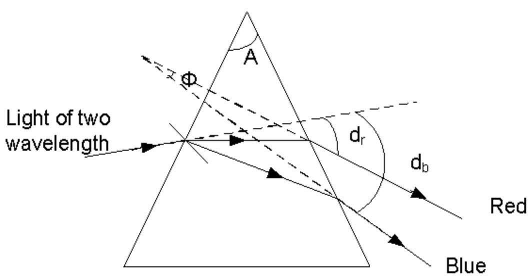
Dispersion of whit light is the separation of white light in to its component colours by a transparent medium due to their speed differences in the medium



When white light falls on a transparent medium, its different component colours travel with different speeds through the medium. They are there fore deviated by different amounts on refraction at the surface of the medium and hence dispersion.

NOTE :

(i) White light is a mixture of various colours. This is called the spectrum of white light.
(ii) The spectrum of white light consists of red, orange, yellow, green ,blue, indigo and violet light bands. On refraction, violet is the most refracted colour away from the normal (violet is the most deviated colour) while red is least deviated
(iii) When light of two wavelengths say red and blue light is incident at a small angle on a small angle prism of refracting angle A having refractive indices of n_r and n_b for the two wave lengths respectively, then the two wave lengths are deviated as shown below.



The deviation of red and blue light is given by $d_r = (n_r - 1)A$ and $d_b = (n_b - 1)A$
The quantity $\phi = d_b - d_r$ is called the **Angular separation (Angular dispersion)** produced by the prism.
$\Rightarrow \phi = (n_r - 1)A - (n_b - 1)A$
on simplifying $\phi = (n_r - n_b)A$

EXAMPLES:

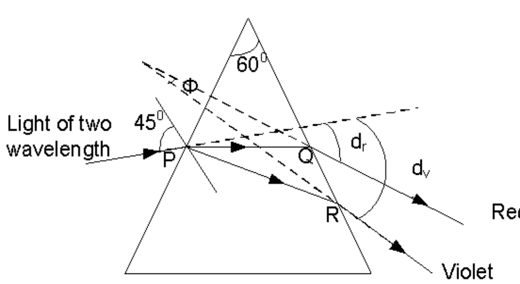
1. Light of two wave length is incident at a small angle on a thin prism of refracting angle 5° and refractive index of 1·52 and 1·48 for the two wave lengths. Find the angular separation of the two wave lengths after refraction by the prism.

Solution

For a small prism,
Angular separation $\phi = (n_1 - n_2)A$

$\phi = (1{\cdot}52 - 1.48)\, x5^{\circ}$
$\Rightarrow \phi = \mathbf{0{\cdot}2}\ ^{\circ}$

2. A glass prism with refracting angle 60° has a refractive index of 1·64 for red light and 1·66 for violet light. Calculate the angular separation of the red and violet rays which emerge from the prism when a ray of white light is incident on the prism at an angle of 45°



Case I: Consider the deviation suffered by red light
At **P,** Snell's law becomes .

$n_a \sin 45^{\circ} = 1{\cdot}64 \sin r_1$
$\therefore\ r_1 = 25{\cdot}54^{\circ}$
But $r_1 + r_2 = 60^{\circ}$
$\Rightarrow r_2 = 60^{\circ} - 25{\cdot}54^{\circ}$
$\therefore\ r_2 = 34{\cdot}46^{\circ}$

At **Q**, Snell's law becomes

$n_a \sin i_2 = 1{\cdot}64 \sin 34{\cdot}46^{\circ}$
$\therefore i_2 = \mathbf{68{\cdot}13^{\circ}}$

Total Deviation $D_r = d_2 + d_1$
where $d_1 = i_1 - r_1$ and $d_2 = i_2 - r_2$
$D_r = (45^{\circ} - 25{\cdot}54^{\circ}) + (68{\cdot}13^{\circ} - 34{\cdot}46^{\circ})$
$\therefore\ D_r = 53{\cdot}13^{\circ}$

Case II: Consider the deviation suffered by violet light
At **P,** Snell's law becomes .

$n_a \sin 45^{\circ} = 1{\cdot}66 \sin r_1$
$\therefore\ r_1 = 25{\cdot}21^{\circ}$
But $r_1 + r_2 = 60^{\circ}$
$r_2 = 60^{\circ} - 25{\cdot}21^{\circ}$
$\therefore\ r_2 = 34{\cdot}79^{\circ}$

At **R**, Snell's law becomes

$n_a \sin i_2 = 1{\cdot}66 \sin\ 34{\cdot}79^\circ$

$\therefore i_2 = \mathbf{71.28^\circ}$.

Total Deviation $\boldsymbol{D_v = d_2 + d_1}$

where $d_1 = i_1 - r_1$ and $d_2 = i_2 - r_2$

$\boldsymbol{D_v} = (45^\circ - 25{\cdot}21^\circ) + (71{\cdot}28^\circ - 34{\cdot}79^\circ)$

$\therefore \boldsymbol{D_v} = 56{\cdot}28^\circ$.

Thus required angular separation $\boldsymbol{\phi = D_v - D_r}$

$\boldsymbol{\phi = 56{\cdot}28^\circ - 53{\cdot}13^\circ}$

$\Rightarrow \quad \boldsymbol{\phi = 3{\cdot}15^\circ}$

APPEARANCE OF WHITE LIGHT PLACED IN WATER

OBSERVATION:

A coloured spectrum is seen inside the water surface with violet on top and red down.

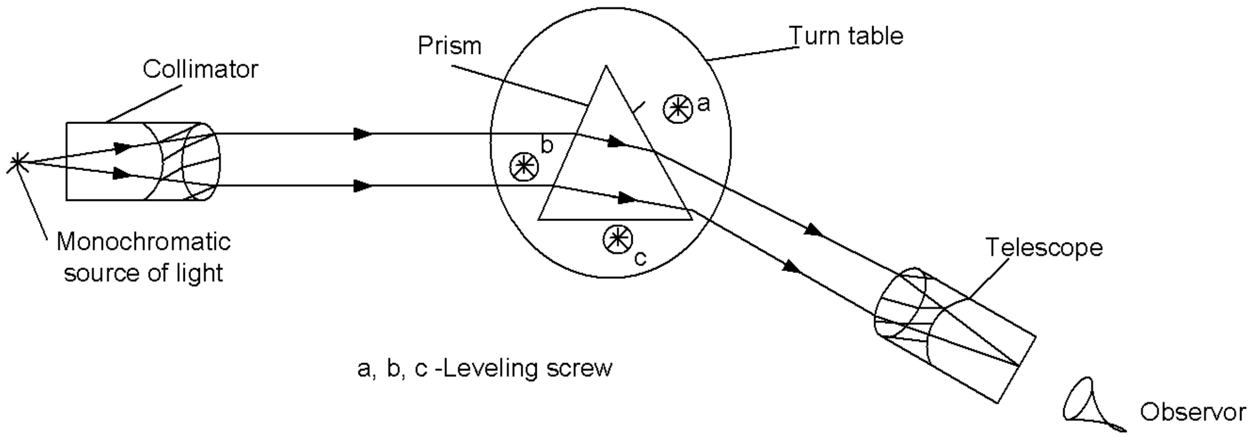
EXPLANATION:

The different component colours of white light travel with different speeds through water. They are there fore deviated by different amounts on refraction at the water surface. Hence different coloured images are formed at different points inside the water surface with a violet coloured image on top.

SPECTROMETER

It is an instrument used to measure accurate determination of deviation of a parallel beam of light which has passed through a prism. This provides a mean of studying optical spectra as well as measuring the angle of prism, minimum deviation and refractive index of glass prism

It consists of a collimator, a telescope, and a turn table on which the prism is placed as shown.



Initial adjustments:

Before the spectrometer is put in to use, 3 adjustments must be made onto it and these include,

(i) The collimator is adjusted to produce parallel rays of light.
(ii) The telescope is adjusted to receive light from the collimator on its cross wire.
(iii) The turn table is leveled.

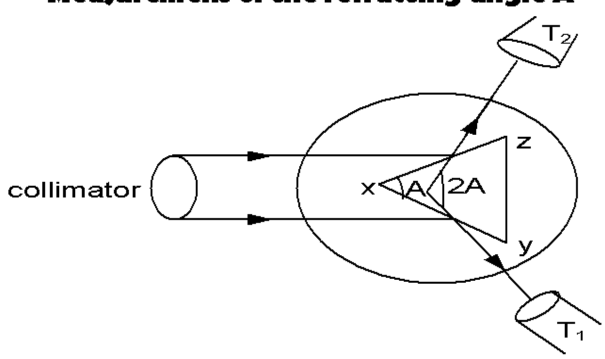
Explanation of the adjustments

i) The eye piece of the telescope is adjusted so that the cross wires are in sharp focus.
The telescope is turned to face a distant object and the length of the telescope is adjusted until the image of the image of the distant object is clearly seen on the cross wires. This means that the telescope receives parallel light.

ii) The prism is removed and the collimator slit is now illuminated using a strong source of monochromatic light. The telescope is now turned to face the collimator and collimator length adjusted until the image of its slit is seen clearly on the cross wires. This means that the collimator is set to produce parallel light.

iii) The prism is now placed on the table. If the image of the slit seen is off the field of view, the screws are adjusted in or out to bring the image to the centre of the field of view. This way the spectrometer is adjusted and ready for use

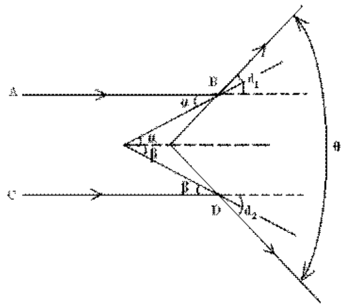
Measurement of the refracting angle A



- The collimator is adjusted to produce parallel rays of light.
- The telescope is adjusted to receive light from the collimator on its cross wire.
- The turn table is leveled
- The prism is placed on the turn table with its refracting angle facing the collimator as shown.
- With the table fixed, the telescope is moved to position T_1 to receive the light from the collimator on its cross wire. This position T_1 is noted
- The telescope is now turned to a new position T_2 to receive light on its cross wire. The angle θ between T_1 and T_2 is measured.
- The prism angle A is given by $A = \frac{\theta}{2}$

PROOF OF THE RELATION

Consider a parallel beam of light incident on to a prism of refracting angle A making glancing angles α and β as shown.



From the geometry, $\alpha + \beta = A$---------.(i).

Deviation d_1 of ray AB $= 2\alpha$

Deviation d_2 of ray CD $= 2\beta$.

Total deviation $\theta = d_1 + d_2$

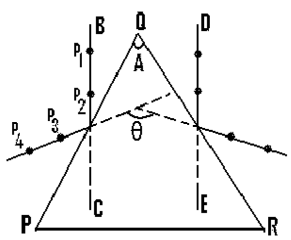
$\theta = 2\alpha + 2\beta$

$\theta = 2(\alpha + \beta)$------------------(ii)

Combining equation (i) and (ii) gives

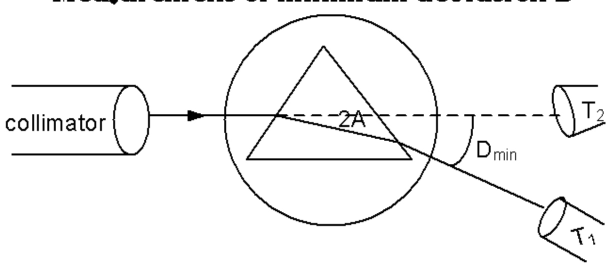
$$\theta = 2A.$$

METHOD 2: USING OPTICAL PINS



- A white paper is stuck to the soft board using top-headed pins. Two parallel line BCand DE are drawn on the paper and the prism is placed with its apex as shown.
- Two optical pins $P_1$and P_2 are placed along BC and pins P_3 and P_4 are placed such that they appear to be in line with the images of P_1 and P_2 as seen by reflection from face PQ .
- The procedure is repeated for face QR.
- The prism is removed and angle θ is measured.
- The required refracting angle $A = \frac{\theta}{2}$

Measurement of minimum deviation D



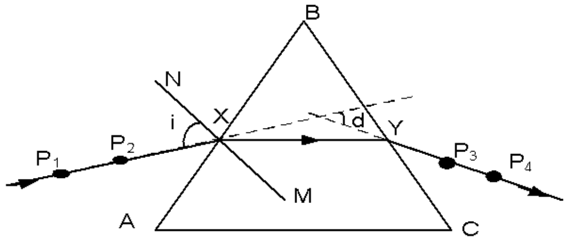
- The collimator is adjusted to produce parallel rays of light.
- The telescope is adjusted to receive light from the collimator on its cross wire
- The turn table is leveled.
- The prism is placed with the refracting angle pointing a way from the collimator as shown above.
- The telescope is turned to receive refracted light form the opposite face of the prism.
- The table is now turned while keeping the refracted light in view until a point when the ray begins to move backwards. Position T_1 of the telescope is noted.
- The prism is removed and the telescope is turned to receive light directly from the collimator. The new position T_2 is marked.
- The angle between T_1 and T_2 is determined and this is the angle of minimum deviation d_{min}.

Note :

The refractive index of the material of the prism is calculated from

$$n = \frac{\sin\left(\frac{d_{min} + A}{2}\right)}{sin\left(\frac{A}{2}\right)}$$

METHOD 2: USING OPTICAL PINS



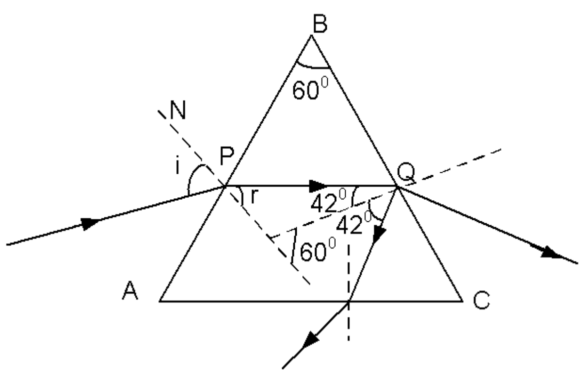
- The prism is palced on a plane sheet of paaper on a soft board and its outline ABC is traced out as shown above.
- A normal NM is drawn through point X on side AB of the prism and a line PX is drawn making an angle i
- Two optical pins P_1 and P_2 are placed along the lines that make different angles of incidence **i.**
- Pins P_3 and P_4 are placed such that they appear to be in line with the images of P_1 and P_2 as seen through the prism.
- The angles of deviation **d** are measured for different angles of incidence.
- A graph of **d** against **i** is plotted to give a curve whose angle of deviation at its turning point is the angle of minimum deviation d_{min} of the prism.

USES OF A GLASS PRISM.

- They enable the refractive index of a glass material to be measured accurately.
- They are used in the dispersion of light emitted by glowing objects.
- They are used as reflecting surfaces with minimal energy loss.
- They are used in prism binoculars.

More worked out examples

1. A ray of monochromatic light is incident on one face of a glass prism of refracting angle 60° and is totally internally reflected at the next face.
   (i) Draw a diagram to show the path of light through the prism.
   (ii) Calculate the angle of incidence at the first face of the prism if its refractive index is 1·53 and the angle of incidence at the second face is 42°.
   Solution



From the diagram, $r + 42° = 60°$

$\therefore r = 18°$

At P, Snell's becomes

$$n_a \sin i = 1{\cdot}53 \sin 18°$$

$$\therefore i = 28{\cdot}2°$$

NOTE

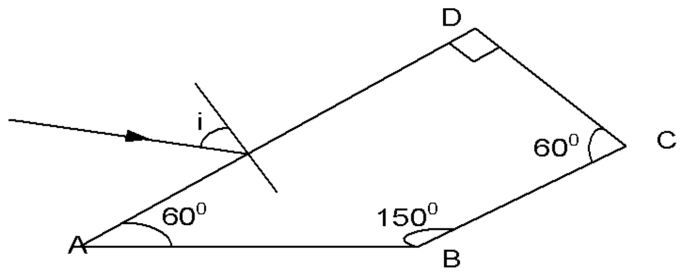
For $n_g = 1{\cdot}53$, then the critical angle c for the above glass material is given by the relation

$$\sin c = \frac{1}{n_g} = \frac{1}{1.53}$$

$$c = 40.8°$$

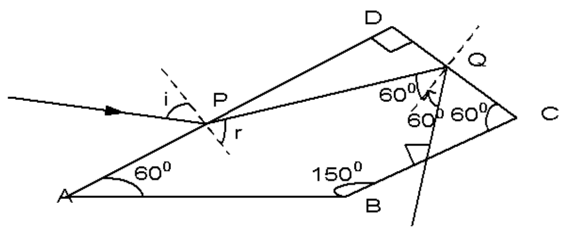
Thus total internal reflection occurs at **Q** since the angle of incidence is greater than the critical angle **c**

2. A ray of light is incident on the face AD of a glass block of refractive index 1·52 as shown.



If the ray emerges normally through face BC after total internal reflection, calculate the angle of incidence, i.

Solution



- After a total internal reflection at Q, t the ray emerges through face BC
- At R, there is no refraction. Therefore Snell's law does not hold at this point.

From ΔRCQ, $\angle RQC + 60^\circ + 90^\circ = 180^\circ$

$\therefore \angle RQC = 30^\circ$

$\Rightarrow$ At Q, the angle of reflection $= 60^\circ$

Hence at Q, the angle of incidence $= 60^\circ$

Solving ΔQDP gives $\angle QPD = 60^\circ$

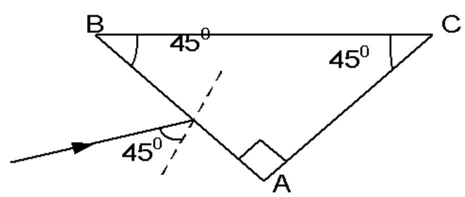
Hence at P, the angle of refraction $r = 30^\circ$

$\Rightarrow$ At P, Snell's law becomes

$$n_a \sin i = 1{\cdot}52 \sin 30^\circ$$

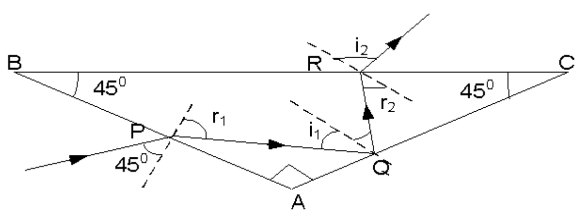
$$\therefore \ i = 49{\cdot}5^\circ$$

3. A ray of light is incident at 45° on a glass prism of refractive index 1·5 as shown.



Calculate the angle of emergence and sketch the ray diagram.

Solution



At P, Snell's becomes $n_a \sin 45^\circ = 1{\cdot}5 \sin r$

$\therefore \ r_1 = 28{\cdot}1^\circ$

At P, $\angle APQ + r_1 = 90^\circ$ where $r_1 = 28{\cdot}1^\circ$

$\Rightarrow$ $\boldsymbol{\angle APQ = 61{\cdot}9^\circ}$

From ΔAPQ, $\angle PQA + 61{\cdot}9^\circ + 90^\circ = 180^\circ$

$\therefore \ \angle PQA = 28{\cdot}1^\circ$

$\Rightarrow$ At Q, the angle of incidence $\boldsymbol{i_1 = 61{\cdot}9^\circ}$

Testing for total internal reflection at Q using the relation

$$\sin c = \frac{1}{n_g} = \frac{1}{1.5}$$

$$c = 41.8°$$

Thus light is totally reflected at **Q** since $\boldsymbol{i_1 > c}$.

$\Rightarrow$ $\boldsymbol{\angle PQA = \angle RQC = 28{\cdot}1^\circ}$

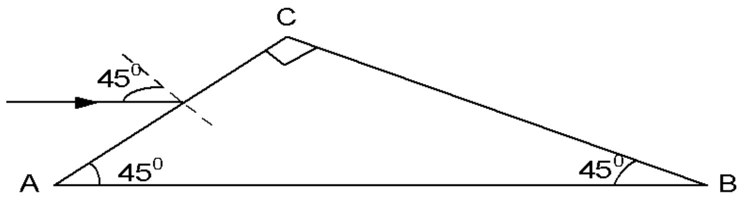
From ΔRQC, $28{\cdot}1^\circ + 45^\circ + 90^\circ + r_2 = 180^\circ$

$r_2 = 16{\cdot}9^\circ$

At R, Snell's becomes $1{\cdot}5 \sin 16{\cdot}9^\circ = n_a \sin i2$

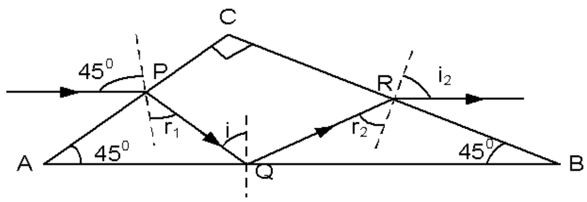
Thus $\boldsymbol{i_2 = 25{\cdot}15^\circ}$

4. A ray of light is incident at 45° on a glass prism of refractive index 1·5 as shown.



Calculate the angle of emergence and sketch the ray diagram.

Solution



At P, Snell's becomes $n_a \sin 45^\circ = 1{\cdot}5 \sin r$

$\therefore r_1 = 28{\cdot}1^\circ$

From ΔAPQ, $\angle PQA + 45^\circ + 90^\circ + r_1 = 180^\circ$

where $r_1 = 28{\cdot}1^\circ$

$\therefore \angle PQA = 16{\cdot}9^\circ$

$\Rightarrow$ At Q, the angle of incidence $i = 73{\cdot}1^\circ$

Testing for total internal reflection at Q using the relation

$$\sin c = \frac{1}{n_g} = \frac{1}{1.5}$$

$$c = 41.8^\circ$$

Thus light is totally reflected at **Q** since $i > c$.

$\Rightarrow \angle PQA = \angle RQC = 16{\cdot}9^\circ$

From ΔRQC, $16{\cdot}9^\circ + 45^\circ + 90^\circ + r_2 = 180^\circ$

$r_2 = 28{\cdot}1^\circ$

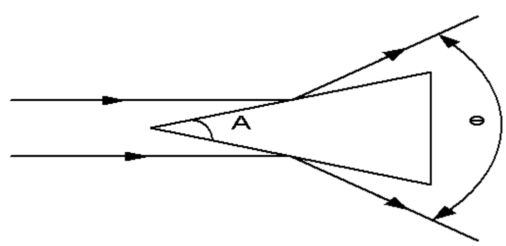
At R, Snell's becomes

$1{\cdot}5 \sin 28{\cdot}1^\circ = n_a \sin i_2$

Thus $i_2 = 45^\circ$

EXERCISE:11

1. Draw a labeled diagram of a spectrometer and State the necessary adjustments that must be made on to it before put in to use.
2. Describe how the refracting angle of the prism can be measured using a spectrometer.
3. You are provided with pins, a white sheet of paper, a drawing board and a triangular prism. Describe how you would determine the refracting angle **A** of the prism
4. A parallel beam of light is incident on to a prism of refracting angle, **A**, as shown



Show that θ = **2A**

5. Describe how the minimum deviation, **D**, of a ray of light passing through a glass prism can be measured using a spectrometer.
6. You are provided with pins, a white sheet of paper, a drawing board and a triangular prism. Describe how you would determine the angle of minimum deviation, **D,** of a ray of light passing through a glass prism.
7. Describe how the refractive index of a material of a glass prism of known refracting angle can be determined using a spectrometer.

8. Describe briefly two uses of glass prisms
9. Light of two wavelengths is incident at a small angle on a thin prism of refracting angle 5° and refractive indices 1.52 and 1.50 for the two wavelengths. Find the angular separation of the two wavelengths after refraction by the prism **An(0.10°)**

REFRACTION THROUGH LENSES

A lens is a piece glass bounced by one or two spherical surfaces.

Types of lenses

There are two types of lenses as shown below

a. Converging (convex) lens

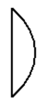
It is a lens which is thicker in the middle than at the edges.

- Its curved outwards
- Thicker in the middle
- Thinner at the edge

Convex lenses are also divided into two namely;

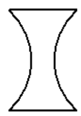
(i) Converging meniscus

(ii) Plano convex



b. Diverging (concave) lens

It is a lens which is thinner in the middle than at the edges.

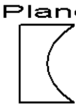


- Its curved inwards
- Thinner in the middle
- Thicker at the edge

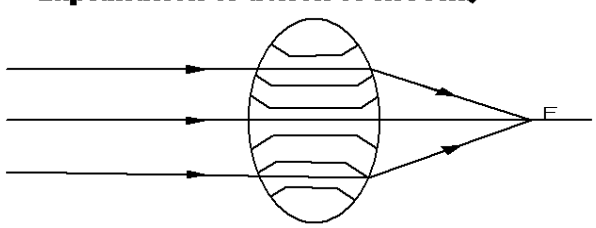
Concave lenses are also divided into two namely;

(i) Diverging meniscus

(ii) Plano concave



Explanation of action of the lens



A thin lens is regarded as made up of a large number of small angle prism whole angles increase from zero at the middle of the lens to a small value at the edge.

The deviation by a small angle prism is $d = (n-1)A$ where n is the refractive index therefore,

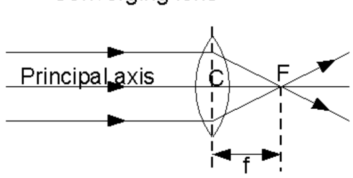
- For light incident on a path of a prism, it bends towards the base and that is why for convex lens, rays converge and for diverging lens, they diverge

Refraction of light in lenses

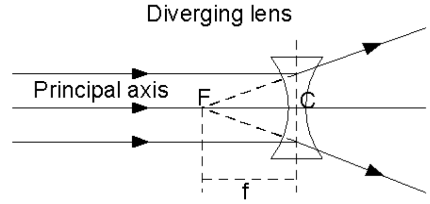
(i) A parallel beam of light, parallel and close to the principal axis of **a converging lens** is converged or brought to focus at the principal focus **F**

(ii) A parallel beam of light, parallel and close to the principal axis of **a diverging lens** is diverged such that the rays appear to come from the principal focus **F**

Converging lens



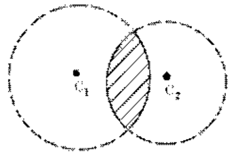
Diverging lens



Terms used in Lenses

Definitions:

1. **Centres of curvature of a lens:** These are centres of the spheres of which the lens surfaces form parts.



Points C_1 and C_2 are the centers of curvature of the lens surfaces.

2. **Radii of curvature of a lens:** These are distances from the centers to the surfaces of the spheres of which the lens surfaces form part.

3. **Principal axis of a lens:** This is the line joining the centers of curvature of the two surfaces of the lens.

4. **Optical centre of the lens:** This is the mid-point of the lens surface through which rays incident on the lens pass un deviated.

5. **Paraxial rays:** These are rays close to the principle axis and make small angles with the lens axis.

6. (i) **Principal focus "F" of a convex lens:** it is a point on the principal axis where where rays originally parallel and closeto the principal axis converge after refraction by the lens.
A convex lens has a real (in front) principal focus.

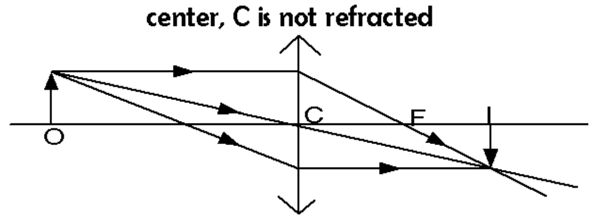
(ii) **Principal focus "F" of a concave lens:** it is a point on the principal axis where where rays originally parallel and closeto the principal axis appear to diverge from after refraction by the lens.
A concave lens has a virtual (behind) principal focus.

7. (i) **Focal length "f" of a convex lens:** it is the distance from the optical centre of the lens to the point where paraxial rays incident and parallel to the principal axis converge after refraction by the lens.
(ii) **Focal length "f" of a concave lens:** it is the distance from the optical centre of the lens to the point where paraxial rays incident and parallel to the principal axis appear to diverge from after refraction by the lens.

Ray diagrams for a converging lens

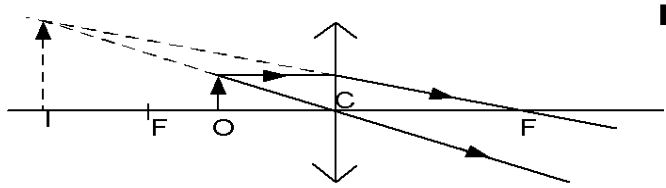
Principal rays for a converging lens

- A ray parallel to the principal axis is refracted to pass through the principal focus F
- A ray passing through the principal focus F is refracted to parallel to the principal axis
- A ray passing through the optical center, C is not refracted



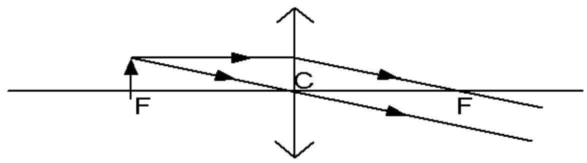
1. **Images formed by a converging lens**

(i) **Object between F and C (Magnifying glass)**



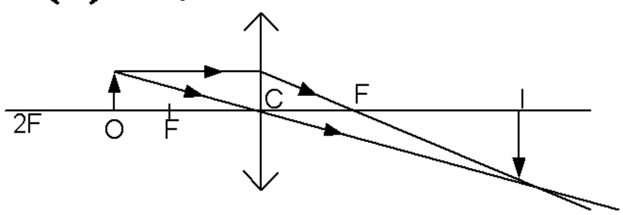
Nature of image
- Virtual
- Erect
- Maginifed

(ii) **Object at F**



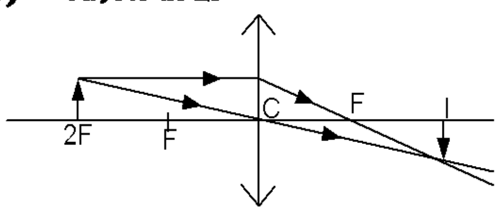
Nature of image
- Image at infinity

(iii) Object between F and 2F



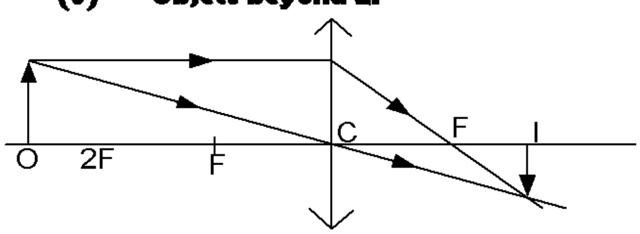
Nature of image
- Real
- Inverted
- Magnified
- Beyond 2F

(iv) Object at 2F



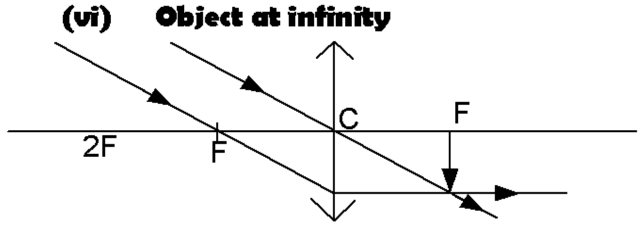
Nature of image
- Real
- Inverted
- Same size as object
- Between F and 2F

(v) Object beyond 2F



Nature of image
- Real
- Inverted
- Diminished
- Between F and 2F

(vi) Object at infinity



Nature of image
- Real
- Inverted
- Diminished
- At F

USES OF CONVEX LENSES

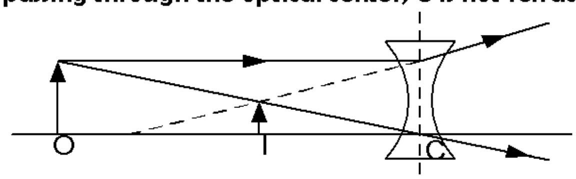
(i) They are used in spectacles for long-sighted people

(ii) They are used in cameras.
(iii) They are used in projectors.
(iv)They are used in microscopes.
(v) They are used in astronomical telescopes.

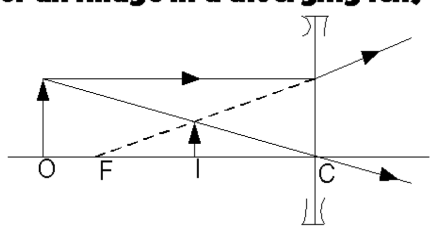
Ray diagrams for a diverging lens

a) **Principal rays for a diverging lens**

- A ray parallel to the principal axis is refracted to appear to come from the principal focus F
- A ray passing through the optical center, C is not refracted



Formation of an image in a diverging lens



Nature of the image

- Virtual
- Erect
- Diminished
- Between F and C

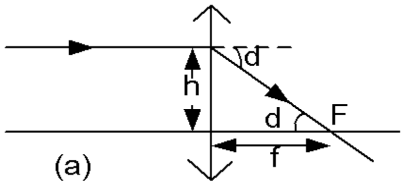
USES OF CONCAVE LENSES

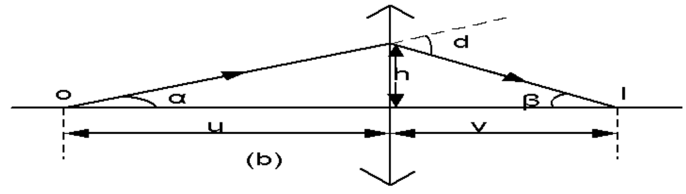
(i) They are used in spectacles for short-sighted people
(ii) They are used in Galilean telescopes.

Thin lens formula

a. Convex lens

Consider array of light incident on a lens close to its principal axis and parallel to it.





From figure (a) above deviation d is small and if it is measured in radians

$$d \approx tan d = \frac{h}{f} \dots\dots\dots\dots(1)$$

From figure (b): $\alpha + \beta = d \dots\dots\dots\dots(2)$

For small angles α, β measured in radians

$$\alpha \approx tan\alpha = \frac{h}{u} \dots\dots\dots\dots(3)$$

$$\beta \approx tan\beta = \frac{h}{v} \dots\dots\dots\dots(4)$$

$$\frac{h}{u} + \frac{h}{v} = \frac{h}{f}$$

From which;

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$f = \frac{u\,v}{u+v}$$

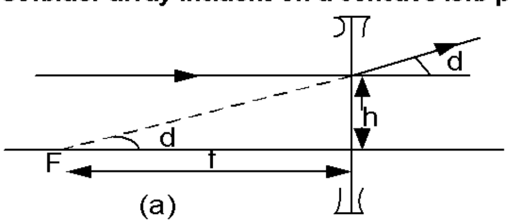
Where
u =object distance
v = image distance
f = focal length

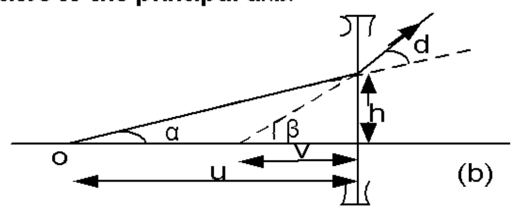
Note:
All distances are measured from the optical center of the lens.

Sign convention

- Distances of real objects and real images are positive ie u and v for real objects and real images are positive.
- Distances of virtual objects and virtual images are negative ie u and v for virtual objects and virtual images are negative.
- Focal length f, for a convex lens is positive and negative for a concave lens.

Concave lens
Consider array incident on a concave lens parallel and close to the principal axis.





From figure (a) above deviation d is small and if it is measured in radians

$$d \approx tand = \frac{h}{f} \ldots\ldots\ldots\ldots (1)$$

From figure (b): $\alpha + d = \beta$

$$-d = (\alpha - \beta) \ldots\ldots\ldots\ldots (2)$$

For small angles α, β measured in radians

$$\alpha \approx tan\alpha = \frac{h}{u} \ldots\ldots\ldots\ldots (3)$$

$$\beta \approx tan\beta = \frac{h}{v} \ldots\ldots\ldots\ldots (4)$$

$$-\frac{h}{f} = \frac{h}{u} - \frac{h}{v}$$

$$-\frac{1}{f} = \frac{1}{u} - \frac{1}{v}$$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

The power of a lenses
The power of a lens,P is the reciprocal of the focal length in metres.
The **S.I unit** of power of a lens is **Dioptres(D)**.
A diopter is the power of a lens of focal length one metre

$$power = \frac{1}{focal\ lenght(metres)}$$

$$\boldsymbol{P = \frac{1}{f(m)}}$$

Examples

(a) Power of a single lens

1. A converging lens has a focal lens 15cm. Calculate the power of the lens.

Solution

$f = 15cm$=0.15m, $P =?$

$$\boldsymbol{P} = \frac{1}{f(m)}$$

$$\mathrm{P} = \frac{1}{0.15}$$

$$\mathrm{P} = 6.67\mathrm{D}$$

2. Find the power of a diverging lens of focal length 10cm

Solution

$f = -10cm = -0.1m, P = ?$

$P = \frac{1}{f(m)}$

$P = \frac{1}{-0.1}$

$P = -`10D$

(b) Power of combination of two lenses in contact

When two lenses are in contact, the power of the combination is obtained by adding the power of the two lenses

$$\boxed{P = P_1 + P_2}$$

1. Two converging lenses of focal length 10cm and 20cm are placed in contact. Find the power of the combination

Solution

For first lens

$f = 10cm = 0.1m, P_1 = ?$

$P_1 = \frac{1}{f(m)}$

$P_1 = \frac{1}{0.1}$

$P_1 = 10D$

For second lens

$f = 20cm = 0.2m, P_2 = ?$

$P_2 = \frac{1}{f(m)}$

$P_2 = \frac{1}{0.2}$

$P_2 = 5D$

Power of the combination

$P = P_1 + P_2$

$P = 10 + 5$

$P = 15D$

2. A converging lens of focal length 10cm is placed in contact with a diverging lens of focal length 25cm. Find the power of the combination

Solution

For first lens

$f = 10cm = 0.1m, P_1 = ?$

$P_1 = \frac{1}{f(m)}$

$P_1 = \frac{1}{0.1}$

$P_1 = 10D$

For second lens

$f = -25cm = -0.25m, P_2 = ?$

$P_2 = \frac{1}{f(m)}$

$P_2 = \frac{1}{-0.25}$

$P_2 = -4D$

Power of the combination

$P = P_1 + P_2$

$P = 10 - 4$

$P = 6D$

LINEAR OR LATERAL MAGNIFICATION

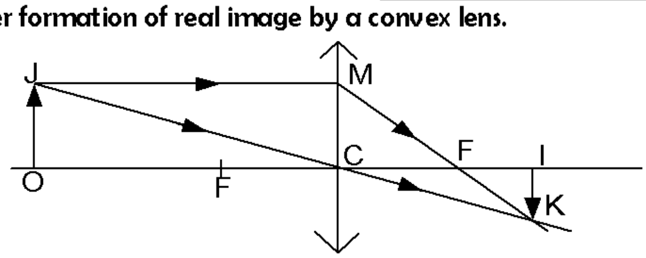
It is defined as the ratio of the image height to object height.

$$\boxed{m = \frac{height\ image}{height\ object}}$$

Magnification is also defined as the ratio of distance of the image from the lens to the distance of the object from the lens.

$$\boxed{m = \frac{image\ distance\ (v)}{object\ distance\ (u)}}$$

Consider formation of real image by a convex lens.



ΔOJC is similar to ΔIKP

$$\frac{IK}{OJ} = \frac{CI}{OC} = \frac{v}{u} = \frac{image\ distance\ (v)}{object\ distance\ (u)}$$

Examples

1. An object is placed 20cm from a converging lens of focal length 15cm. Find the nature, position and the magnification of the image formed

Solution

u = +20cm, f =+15cm, v =?

$i)\ \frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$\frac{1}{15} = \frac{1}{20} + \frac{1}{v}$

$\frac{1}{v} = \frac{1}{15} - \frac{1}{20}$

$\frac{1}{v} = \frac{1}{60}$

v = 60cm

iii) **M =** $\frac{v}{u}$

M = $\frac{60}{20}$

$m = 3$

- Real image(since v = positive)
- Magnified image (since m > 1)

2. A four times magnified virtual image is formed of an object placed 12cm from a converging lens. Calculate;
   (i) The position of the image and
   (ii) The focal length of the lens

Solution

(i) M = 4, u = 12cm, v =?

M = $\frac{v}{u}$

4 = $\frac{v}{12}$

$v = 48cm$

(ii) u =12cm, v = -48cm(virtual image), f=?

$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$\frac{1}{f} = \frac{1}{12} + \frac{1}{-48}$

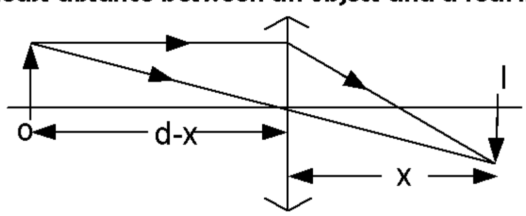
$\frac{1}{f} = \frac{1}{12} - \frac{1}{48}$

$\frac{1}{f} = \frac{3}{48}$

f = 16cm

Least distance between image distance and object distance in a convex lens

Least distance between an object and a real image formed by a convex lens.



Suppose the object and image are a distance d apart with the image a distance x beyond the lens

$u = d - x,\ \ v = x$

$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$\frac{1}{f} = \frac{1}{d-x} + \frac{1}{x}$

$x^2 - dx + df = 0$

$x = \frac{-(-d) \pm \sqrt{(-d)^2 - 4df}}{2}$

For real image $d^2 - 4fd \geq 0$

$d \geq 4f$

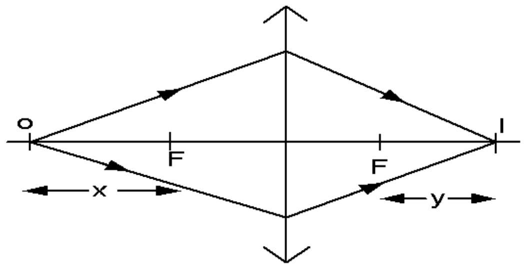
Hence $d = 4f$

The least distance between an object and the real image of the object formed by the convex lens is 4f.

Conjugate points

These are points on the principle axis such that when the object is placed at one , the image is formed at the other.

Suppose a convex lens forms an image of an object O at I ,if the object was placed at I, the lens would form the image of the object at O, then O and I are called conjugate points



$u = f + x\,, \quad v = y + f$

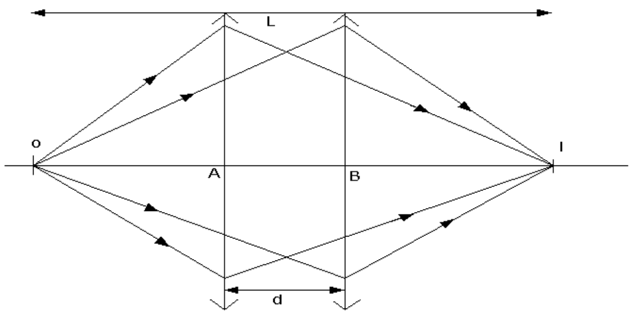
$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{f} = \frac{1}{f + x} + \frac{1}{y + f}$$

$\boxed{f^2 = xy}$ **(Newton's equation)**

Displacement of a lens when an object and screen are fixed.

The lens is moved in position A when the image of O is produced at I, it's a gain displaced through a distance d until the image of O is a gain produced at I.



O and I are conjugate points with respect to the lens.

$OB = AI$ and $OA = BI$

When lens is in position A: $OA + AB + BI = L$

$$u + d + u = L$$

$$u = \frac{l - d}{2}$$

Also: $AI = AB + BI$

$$v = d + u$$

$$v = d + \frac{l - d}{2}$$

$$v = \frac{l + d}{2}$$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

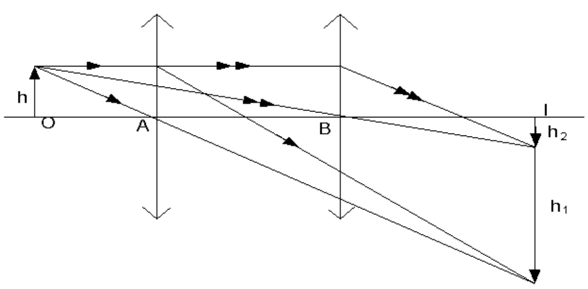
$$\frac{1}{f} = \frac{1}{\left(\frac{l-d}{2}\right)} + \frac{1}{\left(\frac{l+d}{2}\right)}$$

$$\frac{1}{f} = \frac{2}{l - d} + \frac{2}{l + d}$$

$$4lf = l^2 - d^2$$

$$\boxed{f = \frac{l^2 - d^2}{4l}}$$

Displacement of a lens when an erect object and screen are fixed



O and I are conjugate points with respect to the lens.

$OB = AI$ and $OA = BI$

When lens is in position A: $m_A = \frac{AI}{OA} = \frac{h_1}{h}$.........(i)

When lens is in position A: $m_B = \frac{BI}{OB} = \frac{h_2}{h}$........(ii)

But $AI = 0B$

$$\frac{h_1}{h} OA = \frac{h}{h_2} BI$$

$$h^2 = h_1 h_2$$

$$\boxed{h = \sqrt{h_1 h_2}}$$

This method of measuring **h** is most useful when the object is inaccessible for example

(i) When the width of the slit inside the tube is required.

(ii) When the focal length of a thick lens is required.

CONDITION FOR THE FORMATION OF A REAL IMAGE BY A CONVEX LENS

(i) The object distance must be greater than the focal length of the lens.
(ii) The distance between the object and the screen must be at least four times the focal length of the lens.

EXAMPLES:

1. A real image in a converging lens of focal length 15cm is twice as long as the object. Find the image distance from the lens

Solution

$m = \frac{v}{f} - 1$

$2 = \frac{v}{15} - 1$

$v = 45cm$

2. A convex lens of focal length 15cm forms an image three times the height of its object. Find the possible object and corresponding image positions. State the nature of each image.

Solution

Let the object distance = u

$\Rightarrow$ The possible image distances = $+3u$ **or** $-3u$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

Case I where the image distance = $+3u$

$$\frac{1}{15} = \frac{1}{u} + \frac{1}{3u}$$

$\therefore$ The object distance $u = 20cm$

The corresponding image distance:

$3u = 3 \times 20cm = 60cm$

The image is real, magnified, inverted and behind the lens

Case II where the image distance = $-3u$

$$\frac{1}{15} = \frac{1}{u} - \frac{1}{3u}$$

$\therefore$ The object distance $u = 10cm$

The corresponding image distance :

$-3u = -3 \times 10cm = -30cm$

The image is virtual, magnified, erect and same side with the object

3. The magnification of an object in a converging lens is **m**. when the lens is moved a distance **d** towards the object, the magnification becomes m^1 show that the focal length **f** of the lens is given by $f = \frac{dmm^1}{m^1 - m}$.

Solution

Let u be the object distance before displacement

$$\frac{1}{m} = \frac{u}{f} - 1 \ldots\ldots\ldots\ldots\ldots\ldots.(1)$$

Let $u - d$ be the object distance before displacement

$$\frac{1}{m^1} = \frac{u-d}{f} - 1 \ldots\ldots\ldots\ldots\ldots\ldots.(2)$$

(1) −(2)

$$\frac{1}{m} - \frac{1}{m^1} = \frac{d}{f}$$

$$f = \frac{dmm^1}{m^1 - m}$$

4. Two thin convex lenses **A** and **B** of focal lengths **5cm** and **15cm** respectively are placed coaxially **20cm** apart. If an object is placed **6cm** from **A** on the side remote from **B**,

(i) Find the position, nature and magnification of the final image.
(ii) Sketch a ray diagram to show the formation of the final image.

Solution

Consider the action of a convex lens A

$u = 6cm$ and $f = 5cm$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{v} = \frac{1}{5} - \frac{1}{6}$$

$$v = 30cm$$

Consider the action of a convex lens B

The image formed by lens **A** acts as a virtual object for lens **B**.

Thus $u = -(30 - 20)cm = -10cm$ and $f = 15cm$

$$\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$$

$$\frac{1}{v}=\frac{1}{15}-\frac{1}{-10}$$

$$v = 6cm$$

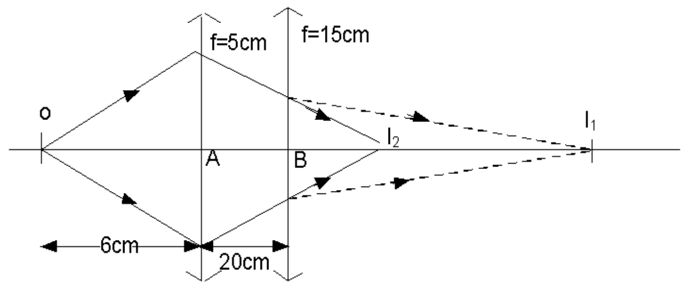
⇒ The image is real and is **6cm** from **B**

$$m = m_1 x m_2$$

$$m = \frac{v_1}{u_1}x\frac{v_2}{u_2}$$

$$m = \frac{30}{6}x\frac{6}{10}$$

$$m = 3$$



5. A thin converging lens **P** of focal length **10cm** and a thin diverging lens **Q** of focal length **15cm** are placed coaxially **50cm** apart. If an object is placed **12cm** from **P** on the side remote from **Q.**
   (i) Find the position, nature and magnification of the final image.
   (ii) Sketch a ray diagram to show the formation of the final image.

Solution

Consider the action of a converging lens P.

$u = 12cm$ and $f = 10cm$

$$\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$$

$$\frac{1}{v}=\frac{1}{10}-\frac{1}{12}$$

$$v = 60cm$$

Consider the action of a diverging lens Q.

The image formed by lens **P** acts as a virtual object for lens **Q.**

Thus $u = -(60 - 50) = -10cm$ and $f = -15cm$

$$\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$$

$$\frac{1}{v}=\frac{1}{-15}-\frac{1}{-10}$$

$$v = 30cm$$

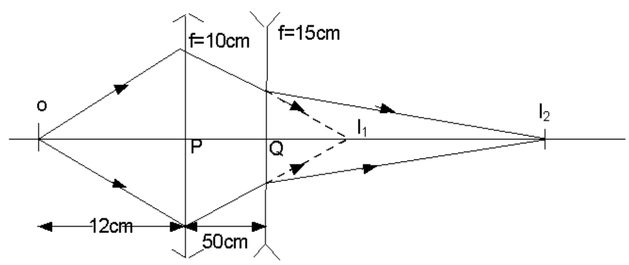
⇒ The image is real and is **30cm** from **Q.**

$$m = m_1 x m_2$$

$$m = \frac{v_1}{u_1}x\frac{v_2}{u_2}$$

$$m = \frac{60}{12}x\frac{30}{10}$$

$$m = 15$$

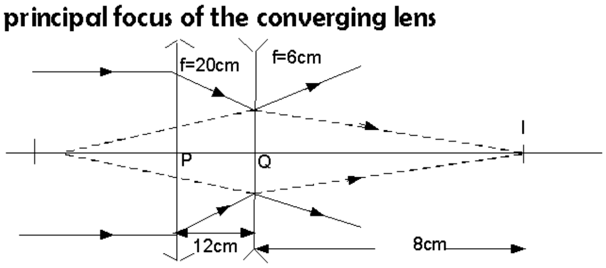


6. Light from a distant object is incident on a converging lens of focal length **20cm** placed **12cm** in front of a diverging lens of focal length **6cm.** Determine the position and nature of the final image.

Solution

Consider the action of a converging lens

The image of a distant object is formed at the principal focus of the converging lens



Consider the action of a diverging lens

The image formed by a converging lens acts as a virtual object for a diverging lens

$\Rightarrow u = -(20 - 12) = -8cm,\ f = -6cm$

$$\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$$

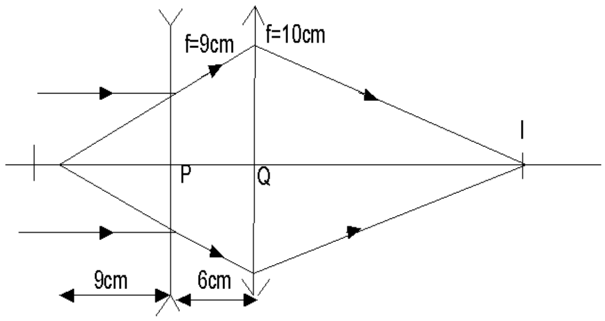
$$\frac{1}{v}=\frac{1}{-6}-\frac{1}{-8}$$

$$v = -24cm$$

⇒ The image is virtual and is 24cm behind the diverging lens.

7. Light from a distant object is incident on a diverging lens of focal length **9cm** placed **6cm** in front of a converging lens of focal length **10cm.** Determine the position and nature of the final image.

Solution



Consider the action of a diverging lens

The image of a distant object is formed at the principal focus of the diverging lens

Consider the action of a converging lens

The image formed by a diverging lens acts as a real object for a converging lens

$\Rightarrow \quad u = (9 + 6)cm = 15cm$ and $f = 10cm$

$$\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$$

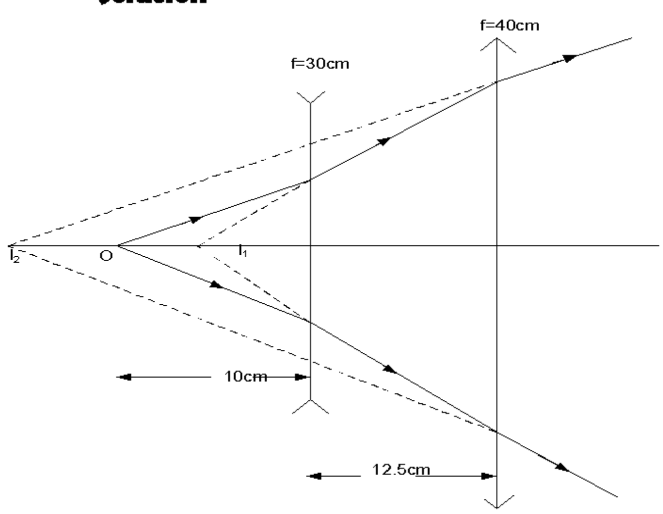
$$\frac{1}{v}=\frac{1}{10}-\frac{1}{15}$$

$$v = 30cm$$

$\Rightarrow$ The image is real and is 30cm from the converging lens.

8. A thin diverging lens of focal length **30cm** and a thin converging lens of focal length **40cm** are placed coaxially **12·5cm** apart. If an object is placed **10cm** from a diverging lens on the side remote from a converging lens**.** Find the position, nature and magnification of the final image.

Solution



Consider the action of a diverging lens

$u = 10cm$, and $f = -30cm$

$$\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$$

$$\frac{1}{v}=\frac{1}{-30}-\frac{1}{10}$$

$$v = 7.5cm$$

Consider the action of a converging lens

The image formed by a diverging lens acts as a real object for a converging lens

$\Rightarrow \quad u = (7.5 + 12.5)cm = 20cm$ and $f = 40cm$

$$\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$$

$$\frac{1}{v}=\frac{1}{40}-\frac{1}{20}$$

$$v = -40cm$$

$\Rightarrow$ The image is virtual and is 40cm from a converging lens.

$$m = m_1 x m_2$$

$$m = \frac{v_1}{u_1}x\frac{v_2}{u_2}$$

$$m = \frac{7.5}{10}x\frac{40}{20}$$

$$m = 1.5$$

9. A luminous object and the screen are placed on a n optical bench and a converging lens is placed between them to show a sharp image of the object on the screen. The linear magnification of the image is found to be 2.5. The lens is now moved 30cm near the screen and a sharp image is again formed on the screen. Calculate the focal length of the lens

Solution

$m = \frac{v}{u}$

$2.5 = \frac{30 + x}{x}$

$x = 20cm$

$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$\frac{1}{f} = \frac{1}{20} + \frac{1}{50}$

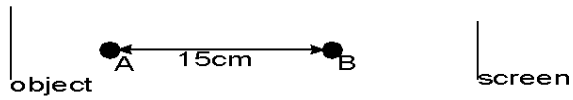
$f = 14.3cm$

Or

$$f = \frac{l^2 - d^2}{4l}$$

$$f = \frac{70^2 - 30^2}{4x70} = 14.3cm$$

10.



In the diagram above the image of the object is formed on the screen when a convex lens is placed either at A or B. If A and B are 15cm apart. Find the;

(i) Focal length of the lens

(ii) Magnification of the image when the lens is at B

Solution

$$f = \frac{l^2 - d^2}{4l}$$

$$f = \frac{45^2 - 15^2}{4x45}$$

$$f = 15cm$$

The magnification m_2 produced by the lens in position **B**

$$m_2 = \frac{l - d}{l + d}$$

$$m_2 = \frac{45 - 15}{45 + 15}$$

$$m_2 = 0.5$$

EXERCISE:11

1. (i) Define the terms centres of curvature, radii of curvature, principal focus and focal length of a converging lens.

(ii) What are **Sign conventions?**

2 (a) An object is placed a distance **u** from a convex lens. The lens forms an image of the object at a distance **v.** Draw a ray diagram to show the path of light when the image formed is:

(i) real

(ii) virtual

(b) Draw a ray diagram to show the formation of an image by a diverging lens.

(c) Draw a ray diagram to show the formation of a real image of a virtual point object by a diverging lens

3. Give two instances in each case where concave lenses and convex lenses are useful.

4. Derive an expression for the focal length **f**, of a convex lens in terms of the object distance **u** and the image distance **v.**

5. Define the term **power of a lens.**

6. (i) Define the term **linear magnification.**

(ii) Show that the linear magnification produced by a convex lens is equal to the ratio of the image distance to the object distance.

(iii) A convex lens of focal length 15cm forms an erect image that is three times the size of the object. Determine the object and its corresponding image position.

(iv) A convex lens of focal length 10cm forms an image five times the height of itsobject. Find the possible object and corresponding image positions.

[**Ans: (iii) u = 10cm, v = –30cm (iv) u = 12cm, v = 60cm OR u = 8cm, v =–40cm**]

7. A convex lens forms on a screen a real image which is twice the size of the object. The object and screen are then moved until the image is five times the size of the object. If the shift of the screen is 20cm, determine the

(i) focal length of the lens

(ii) shift of the object

[**Answers: (i) f = 10cm (ii) 3cm**]

8. A thin converging lens **P** of focal length 2**0cm** and a thin diverging lens **Q** of focal length **30cm** are placed coaxially **0cm** apart. If an object 3cm tall is placed **70cm** from **Q** on the side remote from **Q.**

(i) Find the position final image.

(ii) The height of the final image. **An(60cm, 4.5cm)**

9. A thin converging lens **A** of focal length **6cm** and a thin diverging lens **B** of focal length **15cm** are placed coaxially **14cm** apart. If an object is placed **8cm** from **A** on the side remote from **B,**

(i) Find the position, nature and magnification of the final image.

(ii) Sketch a ray diagram to show the formation of the final image.

[**Answers: (i) 30cm from lens B, real image and magnification = 9**]

10. An object is placed **24cm** in front of a convex lens **P** of focal length **6cm.** When a concave lens **Q** of focal length **12cm** is placed beyond lens **P,** the screen has to be **10cm** away from lens **P** so as to locate the real image formed.

(i) Find the distance between the two lenses **P** and **Q.**

(ii) Sketch a ray diagram to show the formation of the final image.

[**Answer: (i) 4cm**]

11. A lens L_1 casts a real image of a distant object on a screen placed at a distance **15cm** away. When another lens L_2 is placed **5cm** beyond lens L_1, the screen has to be shifted by **10cm** further away to locate the real image formed. Find the focal length and the type of lens L_2.

[**Answer: (i) f = –20 cm and therefore the lens is concave**]

12. A thin convex lens is placed between an object and a screen that are kept fixed **64cm** distant apart. When the position of the lens is adjusted, a clear focused image is obtained on the screen for two lens positions that are **16cm** distant apart.

(i) Draw a ray diagram to show the formation of the images in the two lens positions.

(ii) Find the focal length of the lens

(iii) Find the magnification produced in each lens position.

[**Answers: (ii) f = 15cm (iii)** m_1 **= 1.67 ,** m_2 **= 0.6**]

13. (i) State two conditions necessary for a biconvex lens to form a real image of an object.

(ii) Show that the minimum distance between an object and a screen for a real image to be formed on it is **4f,** where **f** is focal length of a convex lens. Hence show also that the object and its image are then equidistant from the lens.

14. (i) Explain with the aid of a convex lens the term **conjugate foci.**

(ii) What is meant by **reversibility of light** as applied to formation of a real image by a convex lens?

15. A converging lens of focal length **f** is placed between an object and a screen. The position of the screen is adjusted until a clear magnified image is obtained on the screen. Keeping the screen fixed in this position at a distance **L** from the object, the lens is displaced through a distance **d** to obtain a clear diminished image on the screen.

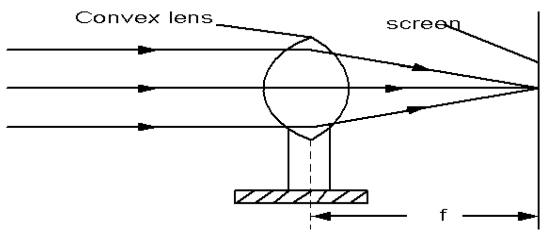
(i) Draw a ray diagram to show the formation of the images in the two cases.

(ii) Show that $l^2 - d^2 = 4lf$

(iii) Find the product of the magnifications produced in the two cases.

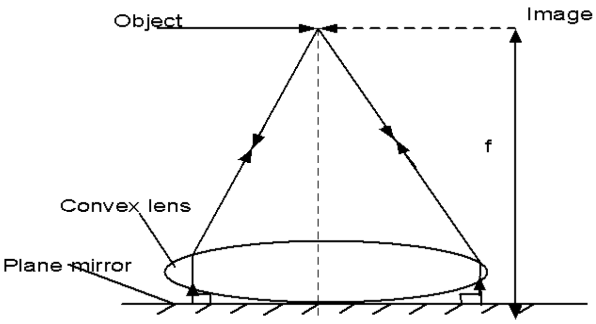
Determining focal length of the converging lens(convex lens)

Method (1) using a distant object



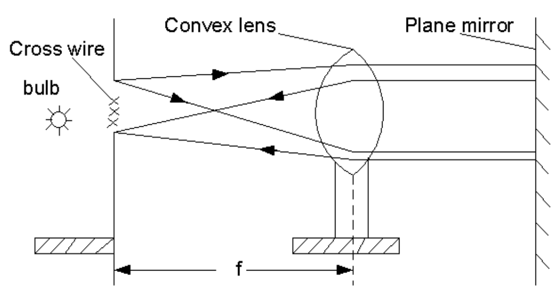
- A distance object such as a window of a tree is focused on to the screen using a convex lens whose focal length is to be determined.
- The distance of the screen from the lens is then the focal length **f** of the lens, which can thus be measured

Method (2) using a plane mirror and the non parallax method.



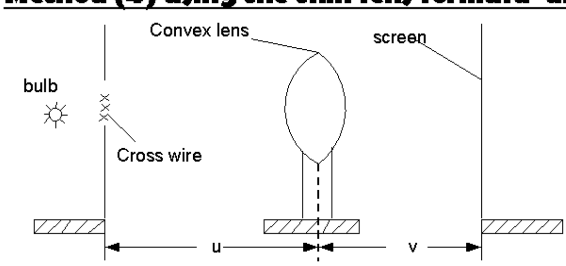
- A plane mirror M is placed on a table and the a biconvex lens is placed on the mirror.
- A pin is clamped horizontally on a retort stand with the apex along the axis of the lens.
- Move the pin up or down to locate the position where the pin coincides with its image using the method of no parallax.
- The distance from the pin O to the lens is measured and this is the focal length ,f, of the lens.

Method (3) using a plane mirror and an illuminated object



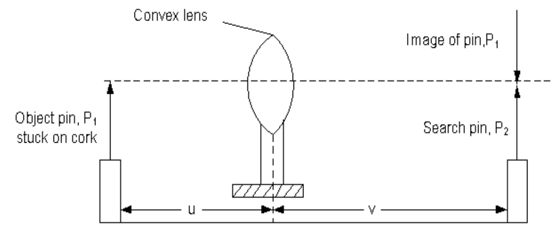
- A lens mounted in a holder is placed between a screen with cross wires and a plane mirror as shown above.
- The lens is moved in between, until a sharp image of the cross-wire is formed on the screen besides the object.
- The distance **f** of the lens from the screen is then the focal length of the lens, which can thus be measured.

Method (4) using the thin lens formula and an illuminated object.



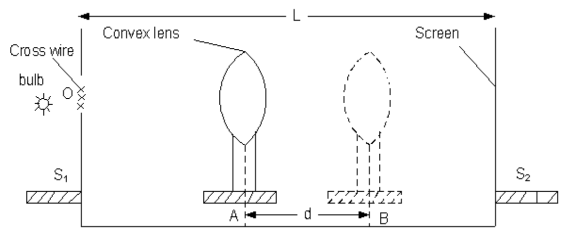
- An illuminated object **O** is placed at a distance **u** in front of a mounted convex lens.
- The position of the screen is adjusted until a sharp image of **O** is formed on the screen at a distance **v** from the lens.
- The procedure is repeated for several values of u and the results are tabulated including values of uv and $u + v$.
- A graph of uv against $u + v$ is plotted and the slope **s** of such a graph is equal to the focal length **f** of the lens.

Method (5) using the thin lens formula and the method of no-parallax.



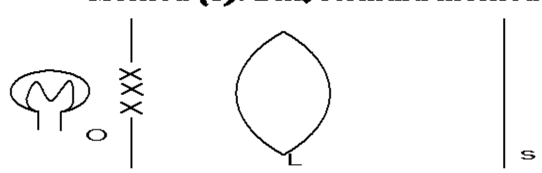
- An object pin P_1 is placed at a distance **u** in front of a mounted convex lens so that its tip lies along the axis of the mirror.
- A search pin P_2 placed behind the lens is adjusted until it coincides with the image of pin p_1 by no-parallax method.
- The distance **v** of pin p_2 from the lens is measured.
- The procedure is repeated for several values of **u** and the results are tabulated including values of uv, and $u + v$.
- A graph of uv against $u + v$ is plotted and the slope **s** of such a graph is equal to the focal length **f** of the lens.

Method (6) using displacement method



- An illuminated object **O** is placed behind the hole in the screen S_1
- The convex lens is placed behind S_2 in a such position **A** so that a <u>sharp magnified image</u> is formed on the screen S_2.
- Keeping the screen and the object fixed, the lens is then moved to position B suc that <u>sharp diminished image</u> is formed on the screen S_2.
- The distance d between A and B is measured
- The distance L between S_1 and S_2 is measured
- The focal length **f** of the lens can then be calculated from $f = \frac{l^2 - d^2}{4l}$

Method (7): Lens formula method



- The apparatus is arranged as above
- The wire gauze is illuminated with a bulb and the position of the lens L is adjusted until the <u>sharp image</u> of the wire gauze is formed on the object screen O
- The <u>distance OL</u> is measured and recorded as u
- The <u>distance LS</u> is measured and recorded as v
- The procedure is repeated for various values of u and v and tabulated including values of $\frac{1}{u}$ and $\frac{1}{v}$
- A graph of $\frac{1}{u}$ and $\frac{1}{v}$ is plotted
- The intercepts A and B are read from the graph and focal length, f is obtained from the equation $f = \frac{2}{A+B}$

NOTE:

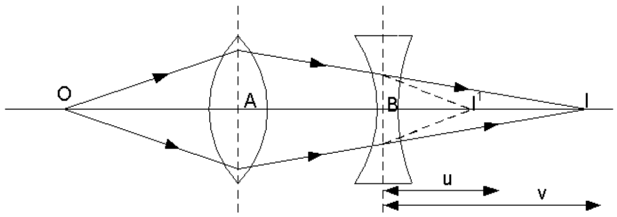
Since no measurements need be made to the surfaces of the lens in Method (5), then it is most useful when finding the focal length of:

(i) a thick lens

(ii) an inaccessible lens, such as that fixed inside an eye-piece or telescope tube.

MEASUREMENT OF FOCAL LENGTH OF A DIVERGING LENS

Method (1) using a converging lens



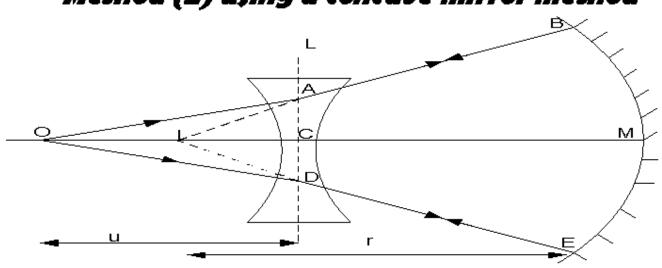
- An object **O** is placed in front of a converging lens of known focal length so as to obtain a real image on the screen at I'
- Measure the distance AI'
- A diverging lens whose focal length is required is placed between the screen and the converging lens.
- The screen is moved to obtain a new real image **I** on to it
- Measure the distance AB and BI
- The object distance u, from this lens is got from $u = -(AI^1 - AB)$.
- The focal length is calculated from $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ where $v = BI$

NOTE:

When a diverging lens is placed in between $\mathbf{L_1}$ and $\mathbf{L_2}$ the image $\mathbf{I_1}$ formed by a converging lens acts as a virtual object for the diverging lens hence giving the final real image **I.**

A virtual object A collection of points which may be regarded as a source of light rays for a portion of an optical system but which does not actually have this function.

Method (2) using a concave mirror method



- An illuminate object O is placed <u>infront</u> of a diverging lens, L **arranged coaxially** with a concave mirror, M of known foval legth, f.
- The postion of object **O** is adjusted until it <u>coincides with its image.</u>
- Measure and record distances **OC** and **CM**
- focal length, f, can be calculated from

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

Where $u = OC$ and $v = -(r - CM)$ **but** $r = 2f$

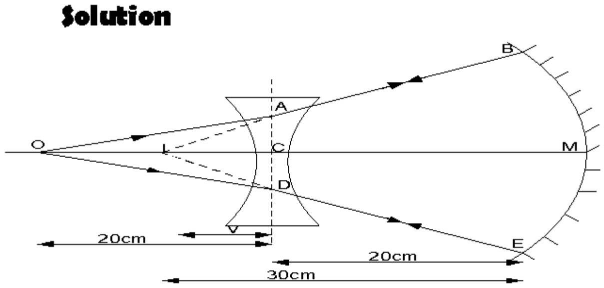
NOTE:

As the object and its image are coincident at **O,** the rays must be incident normally on the mirror and there fore retrace their own path through the centre of curvature of the mirror at **I** and this is the position of the virtual image.

Examples

1. An object is placed 20cm in front of a diverging lens that is coaxially with a concave mirror of focal length 15cm. when concave mirror is 20cm from the lens, the final image coincides with the object
   (i) draw a ray diagram to show how image is formed
   (ii) determine the focal length of the diverging lens

Solution



Consider the action of a diverging lens

$u = 20cm$ and $v = -(30 - 20)cm$

$v = -10cm$ 'virtual image'.

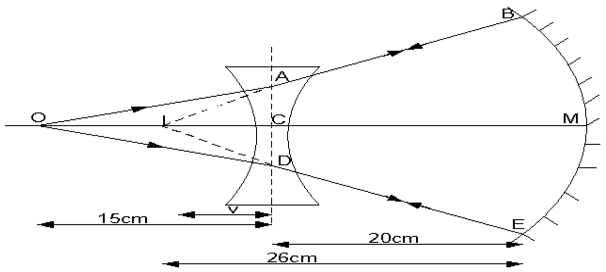
$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{f} = \frac{1}{20} + \frac{1}{-10}$$

$$f = -20cm$$

2. An object is placed **15cm** in front of a diverging lens placed coaxially with a concave mirror of focal length **13cm.** When the concave mirror is **20cm** from the lens the final image coincides with the object.
   (i) draw a ray diagram to show how final image is formed
   (ii) determine the focal length of the diverging lens

Solution



Consider the action of a diverging lens

$u = 15cm$ and $v = -(26 - 20)cm$

$v = -6cm$ 'virtual image'.

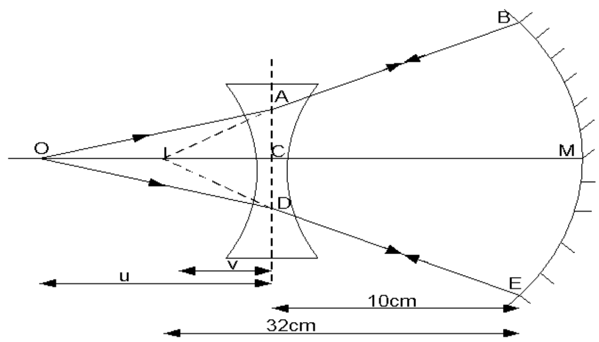
$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{f} = \frac{1}{15} + \frac{1}{-6}$$

$$f = -10cm$$

3. A concave lens of focal length 20cm is placed 10cm in front of concave mirror of focal length 16cm. calculate the distance from the lens at which an object will coincide with its image

Solution



Consider the action of a diverging lens

$f = -20cm$ and $v = -(32 - 20)cm$

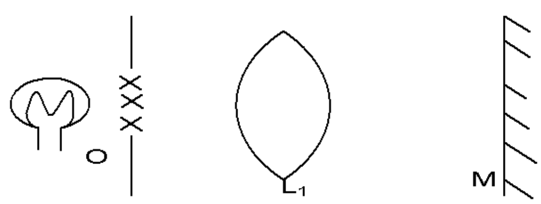
$v = -22cm$ 'virtual image'

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{-20} = \frac{1}{u} + \frac{1}{-22}$$

$$v = -220cm$$

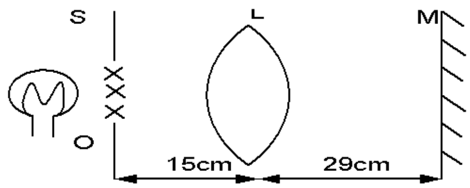
Method (3) Using a plane mirror, converging lens and an illuminated object



- The apparatus is arranged as above
- The wire gauze is illuminated with a bulb and the position of the lens L_1 is adjusted until a sharp image of the wire gauze is formed on the object screen O
- The distance OL_1 is measured and recorded as f_1
- The test lens L_2 is now cemmented on L_1 and again placed between O and M. The position of the combined lens is adjusted until asharp image of the wire gauze is formed at O
- The distance OL_2 is measured and recorded as f
- Focal length of the test lens is then calculated from $\frac{1}{f_2} = \frac{1}{f} - \frac{1}{f_1}$

Examples

A convex lens **L**, a plane mirror **M** and a screen **S** are arranged as shown below so that a sharp image of an illuminated object **O** is formed on the screen **S**.

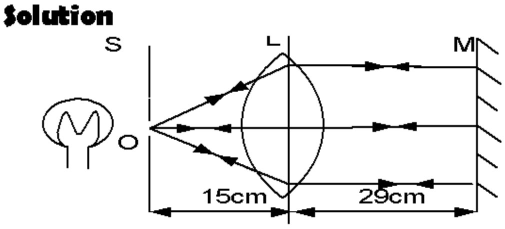


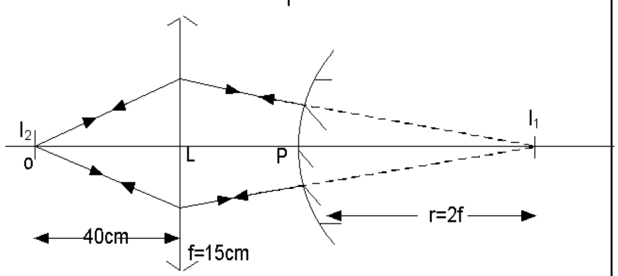
When the plane mirror is replaced by a convex mirror, the lens has to be moved **25cm** towards the mirror so as to obtain a sharp focused image on the screen.

(i) Illustrate the two situations by sketch ray diagrams.

(ii) Calculate the focal length of the convex mirror.

Solution





Consider the action of a convex lens

$u = 40cm$, and $f = 15cm$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{15} = \frac{1}{40} + \frac{1}{v}$$

$$v = 24cm$$

The radius of curvature $r = (24 - 4)cm$

$$r = 20cm$$

Using the relation $r = 2f$

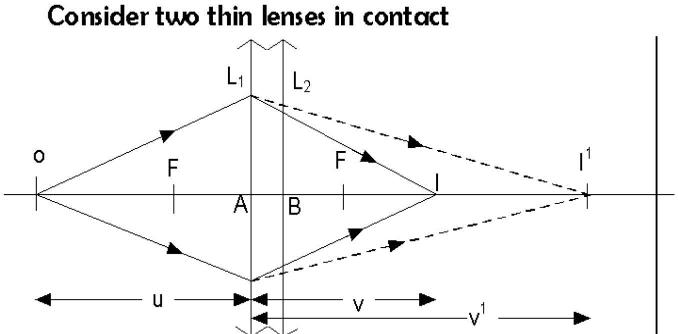
$\Rightarrow \quad 2f = 20cm$

$\therefore \quad f = 10cm$

Thus $f = -10cm$ "The centre of curvature of a convex mirror is virtual"

Combined focal length of two thin lenses in contact

Consider two thin lenses in contact



With lens L_1 of focal length f_1

$$\frac{1}{f_1} = \frac{1}{u} + \frac{1}{v^1} \ldots\ldots\ldots\ldots\ldots (1)$$

The image I^1 forms the virtual object for lens L_1 of focal length f_2

$$\frac{1}{f_2} = -\frac{1}{v^1} + \frac{1}{v} \ldots\ldots\ldots\ldots\ldots (2)$$

equation (1) + (2)

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{u} + \frac{1}{v^1} + \left(-\frac{1}{v^1} + \frac{1}{v}\right)$$

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{u} + \frac{1}{v}$$

But $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ where f is the focal length of the combined lens

$$\boxed{\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}}$$

NOTE:

(i) This formula for f applies for any two lenses in contact such as two diverging lenses or a converging and diverging lens.

(ii) When the formula is used the signs of the focal length must be considered as illustrated below.

Example

1. Suppose a converging lens of focal length 8cm is placed in contact with a diverging lens of focal length 12cm

Solution

$f_1 = +8cm$ and $f_2 = -12cm$

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\frac{1}{f} = \frac{1}{8} + \frac{1}{-12}$$

$$f_2 = 24cm$$

The positive sign shows that the combination acts as a convex lens.

2. Suppose a thin converging lens of focal length 6cm is placed in contact with a diverging lens of focal length 10cm. what is the combined focal length.

Solution

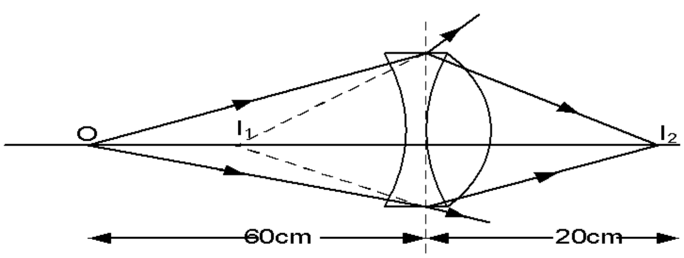
$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\frac{1}{f} = \frac{1}{-10} + \frac{1}{6}$$

$$f_2 = 15cm$$

3. A small object is placed at a distance of 60cm to the left of a diverging lens of focal length 30cm.A converging lens is then placed in contact with the diverging lens. If a real image is formed at a distance of 20 cm to the right of the combined lenses; find the focal length of the converging lens.

Solution



$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

For diverging lens: $u = 60cm, f = -30cm$

$$\frac{1}{-30} = \frac{1}{60} + \frac{1}{v}$$

$v = -20cm$ (virtual image)

For converging lens: virtual image becomes the real object of the converging lens

$$u = 20cm, v = 20cm$$

$$\frac{1}{f} = \frac{1}{20} + \frac{1}{20}$$

$$f = 10cm$$

Or For combination of lenses:

$u = 60cm, v = 20cm$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{f} = \frac{1}{60} + \frac{1}{20}$$

$$f = 15cm$$

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

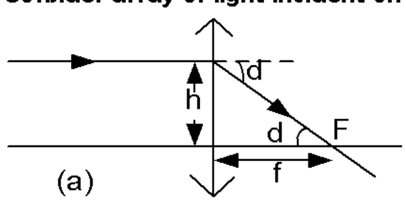
$$\frac{1}{15} = \frac{1}{-30} + \frac{1}{f_2}$$

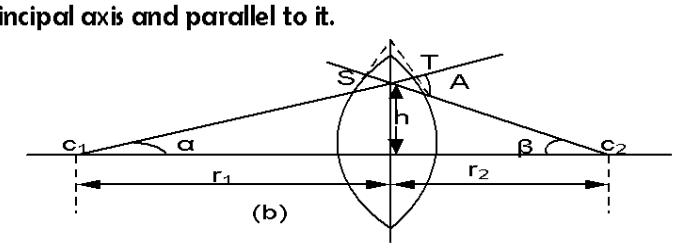
$$f_2 = 10cm$$

Full thin lens formula

Consider the parallel rays RS incident on a convex lens

Consider array of light incident on a lens close to its principal axis and parallel to it.





From figure (a) above deviation d is small and if it is measured in radians

$$d \approx \tan d = \frac{h}{f} \dots\dots\dots\dots(1)$$

for small angle prism d $= (n-1)\,A$

$$\frac{h}{f} = (n-1)\,A \dots\dots\dots\dots(2)$$

For figure (b), the normal at S and T pass through the centre of curvature c_1 and c_2

$\alpha + \beta = A$ (using exterior < properties)

for α and β being small angles in radians,

$$\alpha \approx \tan\alpha = \frac{h}{r_1}$$

$$\beta \approx \tan\beta = \frac{h}{r_2}$$

Hence $\frac{h}{r_1} + \frac{h}{r_2} = A \dots\dots\dots\dots\dots\dots\dots..(3)$

Substitute equation (2) into (3)

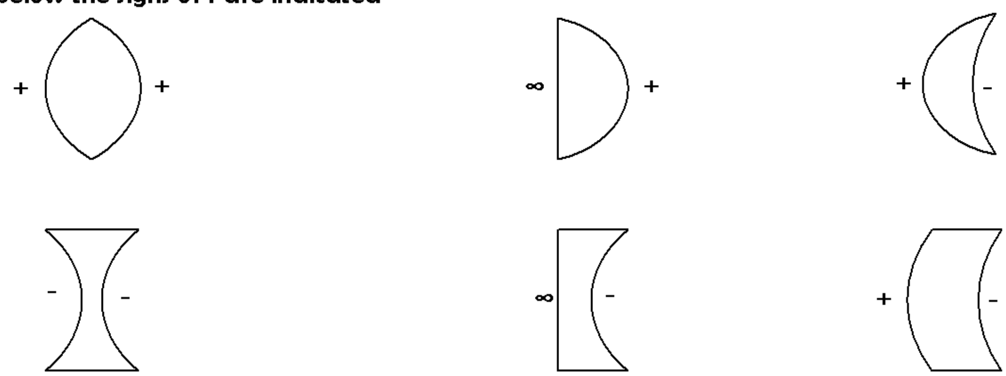
$$\frac{h}{r_1} + \frac{h}{r_2} = \frac{h}{f(n-1)}$$

$$\frac{1}{f(n-1)} = \frac{1}{r_1} + \frac{1}{r_2}$$

Hence $\boxed{\frac{1}{f} = (n-1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)}$

Sign convention for radius of curvature

If the surface of the lens is convex, then the corresponding radii of curvature is positive, however if the surface is concave then its corresponding radius of curvature is negative. Thus for the type of lenses shown below the signs of r are indicated



NOTE:

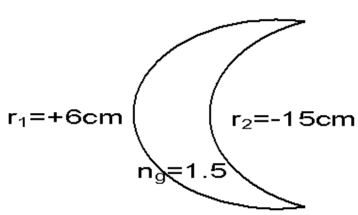
In numerical work for a convex meniscus, the radius of curvature with the largest magnitude takes up the negative sign.

EXAMPLE:

1. A converging meniscus with radii of curvature **15cm** and **6cm** is made of glass of refractive index **1.5.** Calculate its focal length when surrounded by:
   (i) Air
   (ii) a liquid of refractive index **1.2**

Solution:

(i)



$r_1 = 6cm, r_2 = -15cm$ and $n = 1.5$

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$\frac{1}{f} = (1.5-1)\left(\frac{1}{6} + \frac{1}{-15}\right)$$

$$f = 20cm$$

(ii)

$$\frac{1}{f} = (\frac{n_g}{n_l} - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$
$$\frac{1}{f} = (\frac{1.5}{1.2} - 1)\left(\frac{1}{6} + \frac{1}{-15}\right)$$
$$f = 40cm$$

2. A thin lens with faces of radii of curvature 30cm is to be made from the glass with refractive index 1.6. what will be the focal length of the lens if it is;

(i) Biconvex

(ii) biconcave

Solution

(i)

$$\frac{1}{f} = (n - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$
$$\frac{1}{f} = (1.6 - 1)\left(\frac{1}{30} + \frac{1}{30}\right)$$
$$f = 25cm$$

(ii)

$$\frac{1}{f} = (n - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$
$$\frac{1}{f} = (1.6 - 1)\left(\frac{1}{-30} + \frac{1}{-30}\right)$$
$$f = -25cm$$

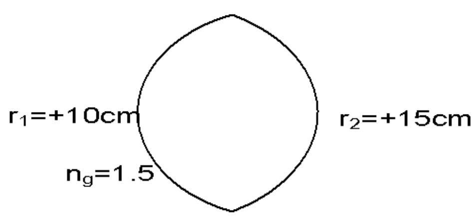
3. Calculate the focal length of converging meniscus with radii 25cmand 20cm whose refractive index is 1.5

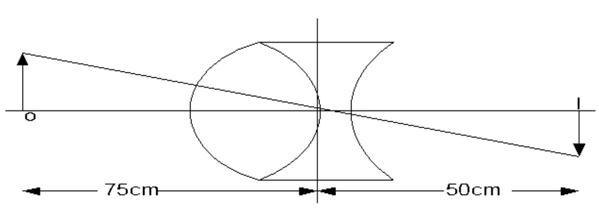
Solution

$$\frac{1}{f} = (n - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$
$$\frac{1}{f} = (1.5 - 1)\left(\frac{1}{25} + \frac{1}{-20}\right)$$
$$f = -200cm$$

4. A thin concave lens is placed in contact with a convex lens made of glass of refractive index **1.5** and its surfaces have radii of curvature **10cm** and **15cm**. If an object placed **75cm** in front of the lens combination gives rise to an image on a screen at a distance **50cm** from the combination, calculate the focal length of the concave lens.

Solution





For length combination $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$$\frac{1}{f} = \frac{1}{75} + \frac{1}{50}$$
$$f = 30cm$$

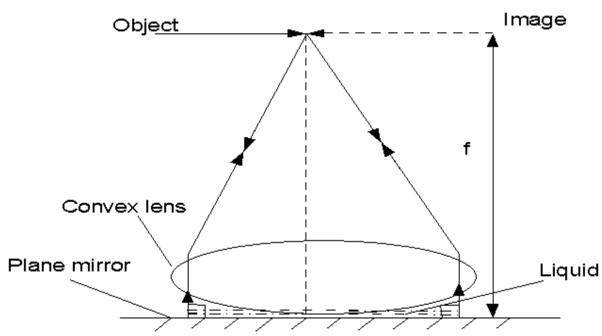
For the convex lens

$$\frac{1}{f_1} = (n_1 - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$
$$\frac{1}{f_1} = (1.5 - 1)\left(\frac{1}{10} + \frac{1}{15}\right)$$
$$f = 12cm$$

But $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$

$$\frac{1}{30} = \frac{1}{12} + \frac{1}{f_2}$$
$$f_2 = -20cm$$

Determining refractive index of a liquid using a convex lens and a plane mirror



- The focal length of the lens is first determined by placing it directly on a plane mirror.
- Clamp a pin horizontally on a retort stand with its apex along the axis of the lens.
- Move the pin up or down to locate the position where the pin coincides with its image using the method of the no parallax.
- Measure the distance of f_1 of the pin from the lens.
- Remove the lens and place a little quantity of the specimen liquid on the plane mirror.
- Replace the convex lens and then locate the new position where the pin coincides with the image.
- Measure the distance f_2 of the pin from the lens. If f_l is the focal length of the liquid then $\frac{1}{f_2} = \frac{1}{f_1} + \frac{1}{f_l}$ and $\frac{1}{f_l} = (n_l - 1)\left(\frac{1}{-r}\right)$
- The refractive index of the liquid is got from $n_l = 1 - \frac{r}{f_2}$
  where r is the radius of curvature of the convex lens

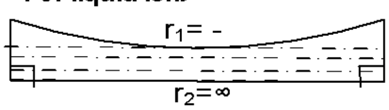
NOTE:
It can be seen from the experiment that the liquid lens is Plano concave type with its lower surface corresponding to the plane surface and the upper surface to the convex lens. There fore $r_1 = -ve$ is the radius of curvature of the upper surface and $r_2 = \infty$ is the radius of curvature of the lower surface.

Example

1. A converging lens is placed on top of a liquid of refractive index 1.4 and a glass slide. Using pin O, position is found where O coincides with it images. If both surfaces of the lens have radii of curvature of 15cm and refractive index of the lens 1.5. Determine the position of coincidence

Solution

For liquid lens

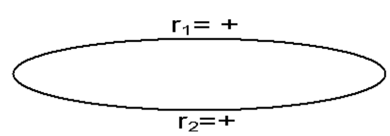


$$\frac{1}{f_l} = (n_l - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$\frac{1}{f_l} = (1.4 - 1)\left(\frac{1}{-15} + \frac{1}{\infty}\right)$$

$$\frac{1}{f_l} = -\frac{0.4}{15}$$

Glass lens



$$\frac{1}{f_g} = (1.5 - 1)\left(\frac{1}{15} + \frac{1}{15}\right)$$

$$\frac{1}{f_g} = \frac{1}{15}$$

For the combination $\frac{1}{f} = \frac{1}{f_l} + \frac{1}{f_g}$

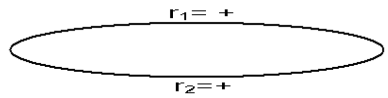
$$\frac{1}{f} = -\frac{0.4}{15} + \frac{1}{15}$$

$$f = 25cm$$

2. A thin equiconvex lens of refractive index 1.5 whose surfaces have radius of curvature 24cm is placed on a horizontal plane mirror. When the space between the mirror and lens is filled with a liquid, a pin held 40cm vertically above the mirror is found to coincide with its own image. What is the refractive index of the liquid

Solution

Glass lens



$$\frac{1}{f_g} = (n_g - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$\frac{1}{f_g} = (1.5 - 1)\left(\frac{1}{24} + \frac{1}{24}\right)$$

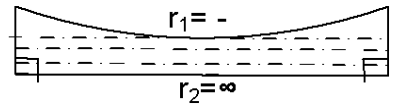
$$\frac{1}{f_g} = \frac{1}{24}$$

For the combination $\frac{1}{f} = \frac{1}{f_l} + \frac{1}{f_g}$

$$\frac{1}{40} = \frac{1}{f_l} + \frac{1}{24}$$

$$\frac{1}{f_l} = -\frac{1}{60}$$

For liquid lens



$$\frac{1}{f_l} = (n_l - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$-\frac{1}{60} = (n_l - 1)\left(\frac{1}{-24} + \frac{1}{\infty}\right)$$

$$n_l = 1.4$$

3. The curved face of a Plano convex lens of refractive index 1.5 is placed in contact with a plane mirror. A pin placed at a distance 20cm coincides with its image. A film of a liquid is now introduced between the lens and the plane mirror. Then the coincidence of the pin and its image is found to be at a distance 100cm. Calculate the refractive index of the liquid.

Solution:

$$\frac{1}{f_g} = (n_g - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$\frac{1}{20} = (1.5 - 1)\left(\frac{1}{24} + \frac{1}{\infty}\right)$$

$$r_2 = 10cm$$

Consider the liquid-lens combination.

$$\frac{1}{f} = \frac{1}{f_l} + \frac{1}{f_g}$$

$$\frac{1}{100} = \frac{1}{f_l} + \frac{1}{20}$$

$$\frac{1}{f_l} = -\frac{1}{25}$$

Consider the liquid lens.

$$\frac{1}{f_l} = (n_l - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$-\frac{1}{25} = (n_l - 1)\left(\frac{1}{-10} + \frac{1}{\infty}\right)$$

$$n_l = 1.4$$

4. A small quantity of a liquid of refractive index 1.4 is poured on a horizontal plane mirror and a biconvex lens of focal length 30cm and refractive index 1.5 is then placed on top of the liquid. The pin is moved along the axis of the lens until no parallax between it and its image find the distance between the pin and the lens.

Solution

$$\frac{1}{f_g} = (n_g - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

For biconvex $r_1 = r_2 = r$

$$\frac{1}{30} = (1.5 - 1)\left(\frac{1}{r} + \frac{1}{r}\right)$$

$$r = 30cm$$

Consider the liquid lens.

$$\frac{1}{f_l} = (n_l - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$\frac{1}{f_l} = (1.4 - 1)\left(\frac{1}{-30} + \frac{1}{\infty}\right)$$

$$f_l = 75cm$$

Consider the liquid-lens combination.

$$\frac{1}{f} = \frac{1}{f_l} + \frac{1}{f_g}$$

$$\frac{1}{f} = \frac{1}{-75} + \frac{1}{30}$$

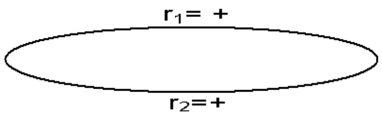
$$f = 50cm$$

The distance between the pin and the lens is **50cm.**

5. **An equi-convex lens of refractive index 1.5 is placed on a horizontal plane mirror. A pin coincides with its own image when it is 0.6m above the lens. When the space between the mirror and lens is filled with a liquid, a pin has to be raised by 0.20m for coincidence to occur again. What is the refractive index of the liquid**

Solution

Glass lens



$$\frac{1}{f_g} = (n_g - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$\frac{1}{60} = (1.5 - 1)\left(\frac{1}{r} + \frac{1}{r}\right)$$

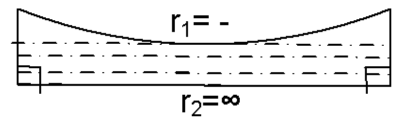
$$r = 60cm$$

For the combination $\frac{1}{f} = \frac{1}{f_l} + \frac{1}{f_g}$

$$\frac{1}{80} = \frac{1}{f_l} + \frac{1}{60}$$

$$\frac{1}{f_l} = -\frac{1}{240}$$

For liquid lens



$$\frac{1}{f_l} = (n_l - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

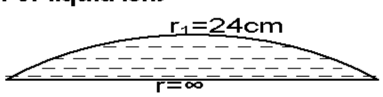
$$-\frac{1}{240} = (n_l - 1)\left(\frac{1}{-60} + \frac{1}{\infty}\right)$$

$$n_l = 1.25$$

6. **A biconcave lens of radius of curvature 24cm is placed on a liquid film on a plane mirror. A pin clamped horizontally above the lens coincides with image at a distance of 40cm above the lens. If the refractive index of the liquid is 1.4 ,what is the refractive index of the material of the lens.**

Solution

For liquid lens



$$\frac{1}{f_l} = (n_l - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$\frac{1}{f_l} = (1.4 - 1)\left(\frac{1}{24} + \frac{1}{\infty}\right)$$

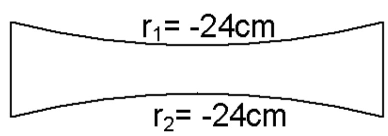
$$\frac{1}{f_l} = \frac{1}{60}$$

For the combination $\frac{1}{f} = \frac{1}{f_l} + \frac{1}{f_g}$

$$-\frac{1}{40} = \frac{1}{f_g} + \frac{1}{60}$$

$$\frac{1}{f_g} = -\frac{1}{24}$$

Glass lens



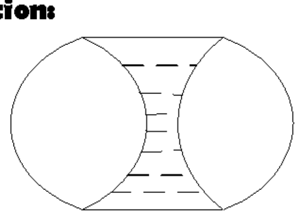
$$\frac{1}{f_g} = (n_g - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$-\frac{1}{24} = (n_g - 1)\left(\frac{1}{-24} + \frac{1}{-24}\right)$$

$$n_g = 1.5$$

8. **Two equiconvex lenses of focal length 20cm and made of glass of refractive index 1·6 are placed in contact and the space between them is filled with a liquid of refractive index 1·4. Find the focal length of the lens combination.**

Solution:



$$\frac{1}{f_g} = (n_g - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

For biconvex $r_1 = r_2 = r$

$$\frac{1}{20} = (1.6 - 1)\left(\frac{1}{r} + \frac{1}{r}\right)$$

$$r = 24cm$$

Consider the equi concave lens.

$$\frac{1}{f_l} = (n_l - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$\frac{1}{f_l} = (1.4 - 1)\left(\frac{1}{-24} + \frac{1}{-24}\right)$$

$$f_l = -30cm$$

Consider the liquid-lens combination.

$$\frac{1}{f} = \frac{1}{f_l} + \frac{1}{f_g} + \frac{1}{f_g}$$

$$\frac{1}{f} = \frac{1}{-30} + \frac{1}{20} + \frac{1}{20}$$

$$f = 15cm$$

focal length of the lens combination **15cm.**

9. In an experiment to determine the refractive index of paraffin the apparatus was first set up as shown using a convex lens of focal length f**:** Some water of refractive index $^4/_3$ was placed on the mirror and the lens on top. A pin placed at a height h_1 vertically above the lens coincides with its image. The experiment was repeated using paraffin instead of water and the new position of coincidence was found to be at a height h_2. Show that the refractive index n_p of paraffin is given by

$$n_p = 1 + \frac{h_1(h_2 - f)}{3h_2(h_1 - f)}$$

Solution

Consider the liquid-lens combination.

$$\frac{1}{f} = \frac{1}{f_l} + \frac{1}{f_g}$$

$\frac{1}{h_1} = \frac{1}{f_l} + \frac{1}{f}$.......................(1)

But

$$\frac{1}{f_l} = (n_l - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$\frac{1}{f_l} = \left(^4/_3 - 1\right)\left(\frac{1}{-r_1} + \frac{1}{\infty}\right)$$

$$\frac{1}{f_l} = \frac{1}{-3r_1}$$

put into (1) $\frac{1}{h_1} = \frac{1}{f_l} + \frac{1}{f}$

$$\frac{1}{h_1} = \frac{1}{-3r_1} + \frac{1}{f}$$

$r_1 = \frac{h_1 f}{3(h_1 - f)}$.......................(2)

Consider the paraffin-lens combination.

$\frac{1}{h_2} = \frac{1}{f_p} + \frac{1}{f}$.......................(3)

But

$$\frac{1}{f_p} = (n_p - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$\frac{1}{f_p} = (n_p - 1)\left(\frac{1}{-r_1} + \frac{1}{\infty}\right)$$

$$\frac{1}{f_p} = \frac{1 - n_p}{r_1}$$

put into (3) $\frac{1}{h_2} = \frac{1}{f_p} + \frac{1}{f}$

$$\frac{1}{h_2} = \frac{1 - n_p}{r_1} + \frac{1}{f}$$

$n_p = 1 + \left(\frac{h_2 - f}{h_2 f}\right) r_1$.......................(4)

Putting (2) into (4)

$$n_p = 1 + \left(\frac{h_2 - f}{h_2 f}\right)\left(\frac{h_1 f}{3(h_1 - f)}\right)$$

$$n_p = 1 + \frac{h_1(h_2 - f)}{3h_2(h_1 - f)}$$

DEFECTS IN IMAGES (ABERRATIONS)

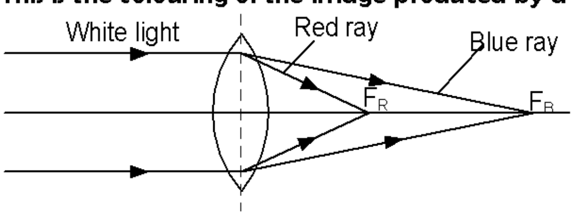
This is the distortion of images formed by either spherical mirrors or spherical lenses.

When mirrors and lenses under consideration are of large aperture, images formed by them can differ in shape and color from the object. Such defects are known as aberration or defects in images. There are two types of aberrations namely:

(i) Chromatic aberration.
(ii) Spherical aberration.

CHROMATIC ABERRATION

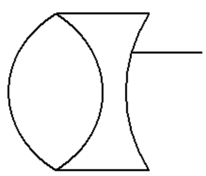
This is the colouring of the image produced by a lens



- When white light is incident on a lens, the different colour components are refracted by different amounts.
- Images corresponding to the different colours are formed in different positions along the principal axis of the lens.
- The image viewed has colored edges.

Minimizing chromatic aberration

Chromatic aberration can be reduced by using an achromatic doublet. This consists of a convex lens combined with a concave lens made from different glass materials. The convex lens deviates the rays while the concave lens nullifies the diversion.



NOTE:

The distance $F_r\ F_v$ **(i.e.** $F_r - F_v$**)** is the longitudinal chromatic aberration for the lens.

Conditions for chromatic doublet to work

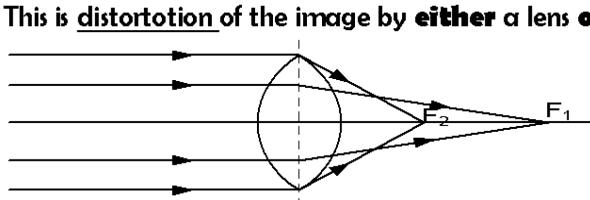
- Lenses should be of different glasses eg crown and flint glasses
- Ratio of their focal length should be equal to the ratio of their dispersive power
- If lenses are of same glass then should not be in contact with each other
  The separation between them should be equal to mean of their focal lengths

This is as a result of the marginal rays being converged nearer the lens or mirror than the paraxial rays.

In lenses, Spherical aberration can be reduced by using a circular stop to cut off marginal rays.

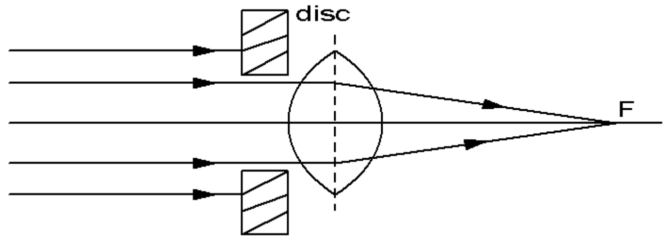
SPHERICAL ABERRATION

This is distortotion of the image by **either** a lens **or** a mirror of wide aperture.



When a wide beam of white light is incident on **either** a lens **or** a mirror of wide aperture, central rays are brought to converge far away from the lens. The rays which are far from the principal axis are brought to converge near the lens. The image formed is circular blurred due to a series of images of the same object

In lenses, Spherical aberration can be minimized using a stopper i.e. using an opaque disc with a central hole to cut off marginal rays.



The disadvantage with this method is light intensity is cut down and so the brightness of the image is reduced.

NOTE:
A circular stop is an opaque disc having a hole in the middle for allowing in only paraxial rays incident on the lens
We can also use plano convex lens with curved side facing the incident rays

In mirrors, spherical aberration can be minimized by using a parabolic mirror.

This because a parabolic mirror converges a wide parallel beam of light incident onto its surface to a single focus as shown.

COMPARISION OF NARROW AND WIDE APERTURE LENSES
Lenses of narrow aperture are widely used in optical instruments so as to avoid spherical aberration. This is because when a wide beam of light falls on a lens of narrow aperture, all rays are paraxial and are thus brought to a single focus to form a sharp image. However a lens with a wide aperture allows in both paraxial and marginal rays, which are thus brought to different focus to form a blurred image.

EXAMPLE:

1. The curved surface of a plane convex lens has a radius of curvature of **20cm** and is made of crown glass for which the refractive index of red and blue light respectively is 1·5 and 1·52 calculate the longitudinal chromatic aberration for the lens.

Solution

For red light $\frac{1}{f_r} = (n_r - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$

$$\frac{1}{f_r} = (1.5 - 1)\left(\frac{1}{20} + \frac{1}{\infty}\right)$$

$$f_r = 40cm$$

For blue light $\frac{1}{f_b} = (n_b - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$

$$\frac{1}{f_b} = (1.52 - 1)\left(\frac{1}{20} + \frac{1}{\infty}\right)$$

$$f_b = 38.5cm$$

Thus longitudinal aberration $= (F_r - F_v)$

$$= (40 - 38{\cdot}5)cm$$
$$= 1{\cdot}5cm$$

2. A convex lens of radius of curvature **24cm** is made of glass of refractive index for red and violet light of **1·6** and **1·8** respectively. A small object illuminated with white light is placed on the axis of the lens at a distance **45cm** from the lens. Find the separation of the images formed in the red and violet constituents of light.

Solution

$$\frac{1}{f} = (n_g - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

For biconvex $r_1 = r_2 = 24$

For red light $\frac{1}{f_r} = (1.8 - 1)\left(\frac{1}{24} + \frac{1}{24}\right)$

$$f_r = 30cm$$

For violet light $\frac{1}{f_v} = (1.8 - 1)\left(\frac{1}{24} + \frac{1}{24}\right)$

$$f_v = 20cm$$

Also

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

For red light

$$\frac{1}{30} = \frac{1}{45} + \frac{1}{V_r}$$

$$V_r = 90cm$$

For violet light

$$\frac{1}{20} = \frac{1}{45} + \frac{1}{V_V}$$

$$V_V = 36cm$$

Image separation$= V_r - V_V$

$$= 90 - 36$$

$$= 54cm$$

EXERCISE:12

1. Describe how the focal length of a convex lens can be determined using a plane mirror and the non-parallax method.
2. You are provided with the following pieces of apparatus: A screen with cross wires, a lamp, a convex lens, a plane mirror, and a meter ruler. Describe an experiment to determine the focal length of a convex lens using the above apparatus.
3. Describe an experiment, including a graphical analysis of the results to determine the focal length of a convex lens using a no parallax method.
4. Describe an experiment to determine the focal length of a thick convex lens having inaccessible surfaces.
5. A convex lens is contained in a cylindrical tube such that its exact position in the tube is not accessible. Describe how you would determine the focal length of the lens without removing it from the tube
6. Describe how the focal length of a diverging lens can be determined using a convex lens.
7. Describe how the focal length of a concave lens can be obtained using a concave mirror.
8. Derive an expression for the focal length of a combination of two thin converging lenses in contact, in terms of their focal lengths.
9. **(a)** Show that the focal length **f** of a thin convex lens in air is given by
   $\frac{1}{f} = (n - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$, Where **n** is the refractive index of the material of the lens, r_1 and r_2 are the radii of curvature of the surfaces of the lens.
   (b) The radii of curvature of the faces of a thin convex meniscus lens of glass of refractive index **1·75** are **8cm** and **12cm**. Calculate the focal length of the lens when completely surrounded by a liquid f refractive index **1·25.**[**Answer: f = 60cm**]
10. **(a)** Describe, giving the relevant equations, how the refractive index of a liquid can be determined using a convex lens of known radius of curvature.
    (b) A biconvex lens of radius of curvature **24cm** is placed on a liquid film on a plane mirror. A pin clamped horizontally above the lens coincides with its image at a distance of **40cm** above the lens. If the refractive index of the liqui is **1·4**, calculate the refractive index of the material of the lens. [**Answer: n = 1·5**]
11. **(a)** Differentiate between **chromatic** and **spherical aberrations.**
    (b) Explain how the defects in **10(a) above** can be minimized in practice.
    (c) Explain why lenses of narrow aperture are preferred to lenses of wide aperture in optical instruments.

(d) Draw a ray diagram showing the reflection of a wide beam of light by a concave mirror of wide aperture

10. A thin biconvex les is placed on a plane mirror. A pin is clamped above the lens so at it apex lies on the principal axial of the lens. The position of the pin is adjusted until the pin coincides with its image at a distance of 15cm from the mirror. When a thin layer of water of refractive index 1.33 is placed between the mirror ,the pin coincides with its image at a point 22.5cm from the mirror. When water is replaced by paraffin, the pin coincides with the image at a distance of 27.5cm from the mirror. Calculate the refractive index of paraffin. **An(1.45)**
11. A bi-convex lens of radius of curvature 24cm is placed on a liquid film on a plane mirror. Apin clamped horizontally above the lens coincides with its image at a distance of 40cm above the lens. If the refractive index of the liquid is 1.4. What is the refractive index of the material of the lens **An(1.5)**
12. An equi-convex lens is placed on a horizontal palne mirror and a pin held vertically above the lens is found to coincide with its image when positioned 20.0cm above the lens. When a few drops of liquid is placed between the lens and the mirro, the pin has to be raised bu 10.0cm to obtain coincidence with the image. If the refractive index of converging lens. What is the refractive index of the liquid **An(1.33)**
13. An illuminated object is placed on the 0cm mark of an optical bench. A converging lens of focal length 15cm is placed at the 22.5cm mark. A diverging lens of focal length 30cm and a plane mirror are placed at the 37.5cm and 77.5cm marks respectively. Find the position of the final image.(at 0cm mark). Illustrate your answer with a ray diagram
14. A converging lens of focal length 1.5cm is placed 29.0cm in front of another converging lens of 6.25cm.An object of height 0.1cm is placed 1.6cm a way from the first lens on the side remote from the second lens at right angles to the principal axis of the final image by the system. Determine the position and size and final image of the object.
15. An equi-convex lens A is made of glass of refractive index 1.5 and has a power of 5.0radm^{-1}. It is combined in contact with a lens B to produce a combination whose power is 1.0radm^{-1}. the surfaces in contact fits exactly. The refractive index of the glass in lens B is 1.6. What are the radii of the four surfaces? Draw a diagram to illustrate your.
16. A lens forms the image of a distant object on a screen 30cm away. Where should a second lens of focal length 30cm be placed so that the screen has to be moved 8cm towards the first lens for the new image to be in focus.
17. A convex lens of focal length 20cm, forms an image on a screen placed 40cm beyond the lens . A concave lens of focal length 40cm is then placed between a convex lens and a screen a distance of 20cm from the convex lens.
    (i) Where must the screen be placed in order to receive the new image?
    (ii) What is the magnification produced by the lens system?